

LINES AND ANGLES

4

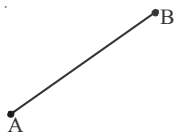
4.0 Introduction

You have learnt some geometrical ideas in previous classes. Let us have fun trying some thing we have already done.



Exercise - 1

1. Name the figures drawn below.



(i)



(ii)



(iii)



(iv)

2. Draw the figures for the following.

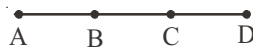
(i) \overline{OP}

(ii) Point X

(iii) \overline{RS}

(iv) \overline{CD}

3. Name all the possible line segments in the figure.

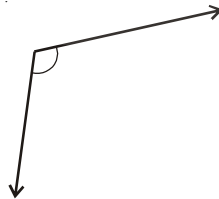


4. Write any five examples of angles that you have observed arround.
Example : The angle formed when a scissor is opened.

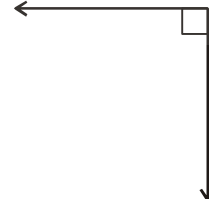
5. Identify the following given angles as acute, right or obtuse.



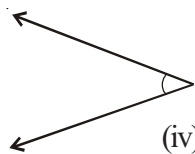
(i)



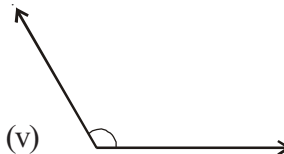
(ii)



(iii)

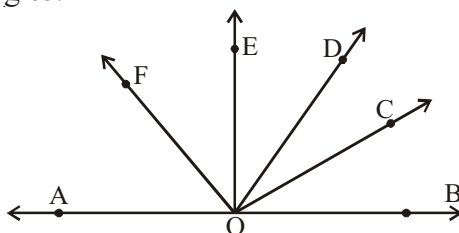


(iv)



(v)

6. Name all the possible angles you can find in the following figure. Which are acute, right, obtuse and straight angles?



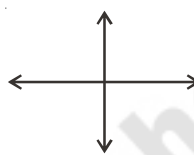
7. Which of the following pairs of lines are parallel? Why?



(i)



(ii)

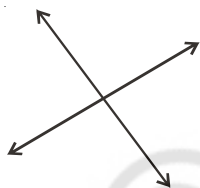


(iii)



(iv)

8. Which of the following lines are intersecting?



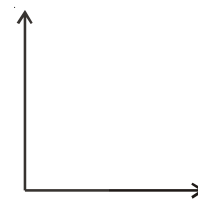
(i)



(ii)



(iii)



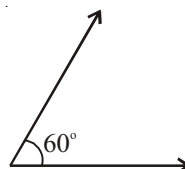
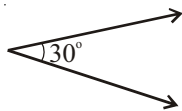
(iv)

4.1 Learning about Pairs of Angles

We have learnt how to identify some angles in the previous chapter. Now we will learn about some more angles as well as various pairs of angles.

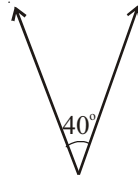
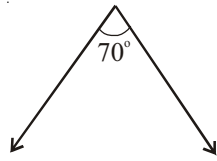
4.1.1 Complementary Angles

When the sum of two angles is equal to 90° , the angles are called complementary angles.



These are complementary angles, as their sum is $30^\circ + 60^\circ = 90^\circ$.

We can also say that the complement of 30° is 60° and the complement of 60° is 30° .



In the above figures, the sum of the two angles is $70^\circ + 40^\circ \neq 90^\circ$. Thus, these angles are not a pair of complementary angles.



Try This

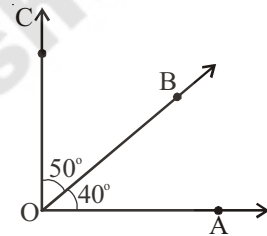
Draw five pairs of complementary angles of your choice.

Do This

Draw an angle $\angle AOB = 40^\circ$. With the same vertex 'O' draw $\angle BOC = 50^\circ$, taking \overrightarrow{OB} as initial ray as shown in the figure.

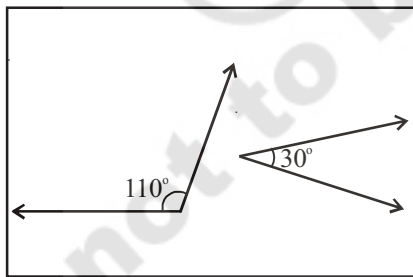
Since the sum of these angles is 90° , they together form a right angle.

Take another pair 60° and 50° and join in the same way. Do they form complementary angles? Why? Why not?

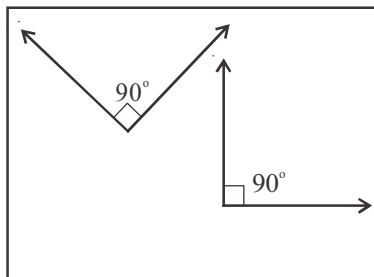


Exercise - 2

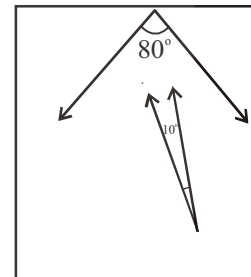
1. Which of the following pairs of angles are complementary?



(i)



(ii)



(iii)

2. Find the complements of the given angles. Find the complementary angles of the following.

(i) 25° (ii) 40° (iii) 89° (iv) 55°

3. Two angles are complement to each other and are also equal. Find them.
4. Manasa says, "Each angle in any pair of complementary angles is always acute". Do you agree? Give reason.

4.1.2 Supplementary Angles

When the sum of two angles are equal to 180° , then the angles are called supplementary angles.



These are a pair of supplementary angles as their sum is $120^\circ + 60^\circ = 180^\circ$.

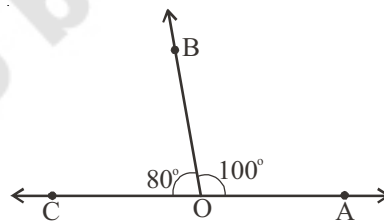
We say that the supplement of 120° is 60° and the supplement of 60° is 120° .



130° and 100° angles are not a pair of supplementary angles. Why?

Do This

Draw an angle $\angle AOB = 100^\circ$. With the same vertex O, draw $\angle BOC = 80^\circ$ such that \overline{OB} is common to two angles.



You will observe that the above angles form a straight angle that is 180° .

You will find that the angles form a straight angle.

Thus, the angles 100° and 80° are supplementary to each other.

Are 130° and 70° supplementary angles? Why? Why not?



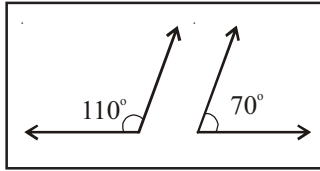
Try This

Write any five pairs of supplementary angles of your choice.

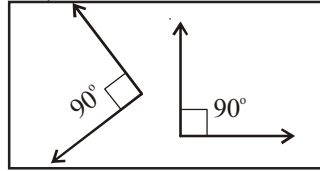


Exercise - 3

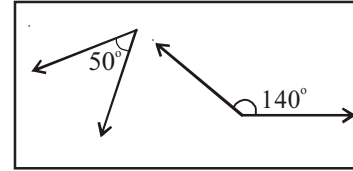
1. Which of the following pairs of angles are supplementary?



(i)



(ii)



(iii)

2. Find the supplementary angles of the given angles.

(i) 105°

(ii) 95°

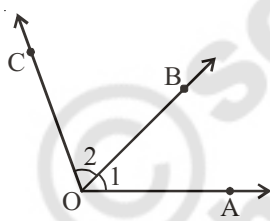
(iii) 150°

(iv) 20°

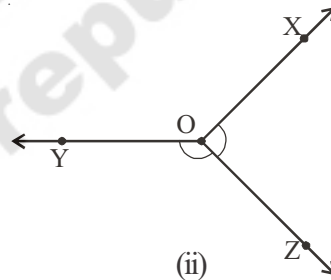
3. Two acute angles cannot form a pair of supplementary angles. Justify.
4. Two angles are equal and supplementary to each other. Find them.

4.1.3 Adjacent Angles

The angles having a common arm and a common 'vertex' are called as adjacent angles.



(i)

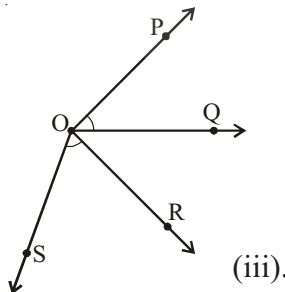


(ii)

The angles $\angle AOB$ and $\angle BOC$ in Figure (i) are adjacent angles, as they have a common vertex 'O' and common arm \overline{OB} .

Are the angles in Figure (ii) adjacent angles? Which is the common vertex and which is the common arm?

Now, look at figure (iii).



(iii).

Are $\angle POQ$ and $\angle ROS$ adjacent angles. Why? Why not?

Which angles are adjacent to each other in the adjacent figure?

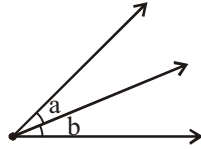
Why do you think they are adjacent angles?



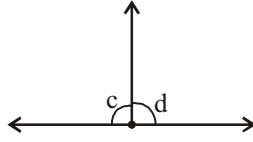


Exercise - 4

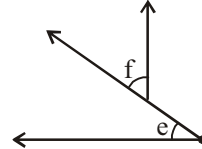
1. Which of the following are adjacent angles?



(i)

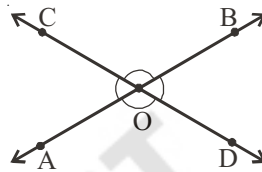


(ii)



(iii)

2. Name all pairs of adjacent angles in the figure. How many pairs of adjacent angles are formed? Why these angles are called adjacent angles?

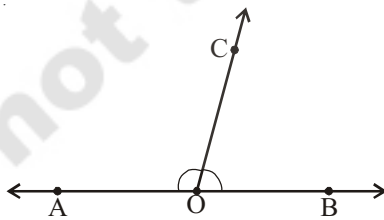


3. Can two adjacent angles be supplementary? Draw figure.
 4. Can two adjacent angles be complementary? Draw figure.
 5. Give four examples of adjacent angles in daily life.

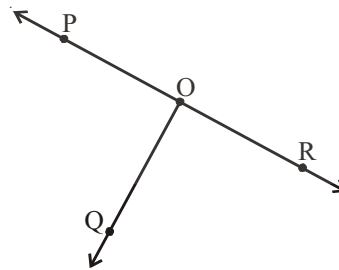
Example : Angles between the spokes at the centre of a cycle wheel.

- (i) _____ (ii) _____
 (iii) _____ (iv) _____

4.1.3 (a) Linear Pair



(i)



(ii)

Look at Figure (i). $\angle AOC$ and $\angle BOC$ are adjacent angles. What is the sum of these angles?

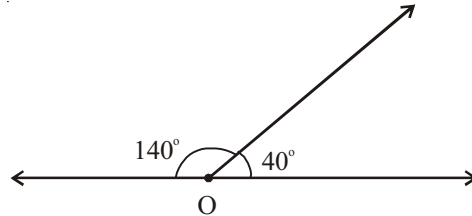
These angles together form a straight angle. Similarly, look at Figure (ii). Do $\angle POQ$ and $\angle ROQ$ together form a straight angle?

A pair of adjacent angles whose sum is a straight angle (or 180°) is called a Linear Pair.

Do This

Two adjacent angles are 40° and 140° . Do they form a linear pair?

Draw a picture and check. Renu drew the picture like this.

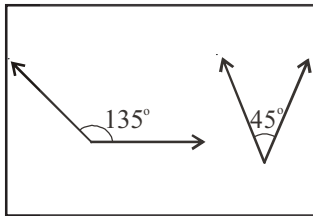


Has she drawn correctly? Do these adjacent angles form a linear pair?

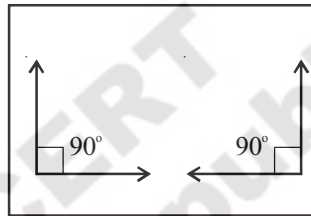


Exercise - 5

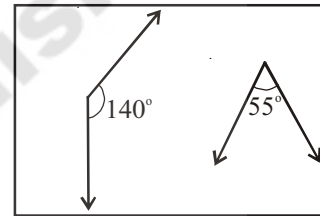
1. Draw the following pairs of angles as adjacent angles. Check whether they form linear pair.



(i)

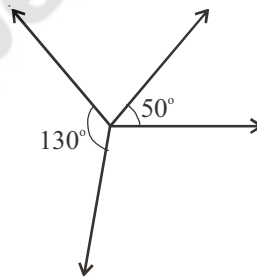


(ii)



(iii)

2. Niharika took two angles- 130° and 50° and tried to check whether they form a linear pair. She made the following picture.

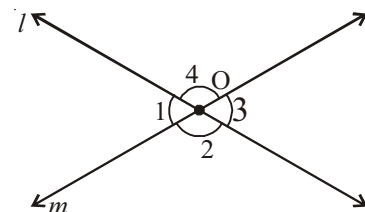


Can we say that these two angles do not form a linear pair? If not, what is Niharika's mistake?

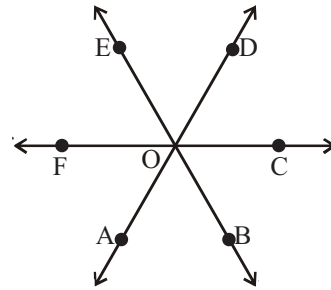
4.1.4 Vertically Opposite Angles

When two lines intersect, the angles that are formed opposite to each other at the point of intersection (vertex) are called vertically opposite angles.

In above figure two lines ' l ' and ' m ' intersect each other at ' O '. Angle $\angle 1$ is opposite to angle $\angle 3$ and the other pair of opposite angles is $\angle 2$ and $\angle 4$. Thus, $\angle 1, \angle 3$ and $\angle 2, \angle 4$ are the two pairs of vertically opposite angles.



Which are the vertically opposite angles in the adjacent figure.



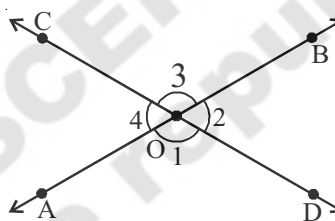
Do This

Draw two lines \overline{AB} and \overline{CD} such that they intersect at point 'O'.

Trace the figure given bellowe on a tracing paper. Place the traced figure over the figure given below and rotate it such that $\angle BOD$ coincides $\angle AOC$. Observe the angles $\angle AOD$ and $\angle BOC$ also $\angle AOC$ and $\angle BOD$.



You will notice that $\angle AOD = \angle BOC$ and $\angle AOC = \angle BOD$.



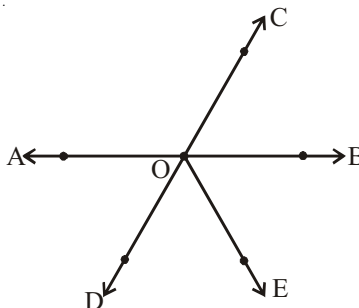
We can conclude that vertically opposite angles are equal.

Note : Take two straws. Fix them at a point 'O' with a pin. Place them such that the straw on top covers the one below. Rotate the straws. You will find that they make vertically opposite angles.

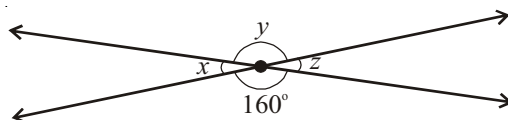


Exercise - 6

1. Name two pairs of vertically opposite angles in the figure.



2. Find the measure of x , y and z without actually measuring them.



3. Give some examples of vertically opposite angles in your surroundings.

4.2 Transversal

You might have seen railway track. This figure is an example when two lines are intersected by a transversal.



A line which intersects two or more lines at distinct points is called a transversal.

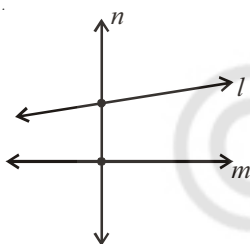


Figure (i)

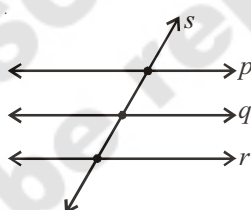


Figure (ii)

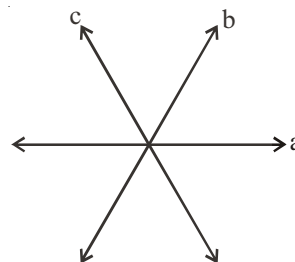


Figure (iii)

In Fig (i) two lines ' l ' and ' m ' are intersected by a line ' n ', at two distinct points.

Therefore, ' n ' is a transversal to ' l ' and ' m '.

In Fig (ii) three lines ' p ', ' q ' and ' r ' are intersected by a line ' s ', at three distinct points.

So, ' s ' is a transversal to ' p ', ' q ' and ' r '.

In Fig (iii) two lines ' a ' and ' b ' are intersected by a line ' c '. The point of intersection of ' c ' is the same as that of ' a ' and ' b '. The three lines are thus intersecting lines and none of them is a transversal to the other.



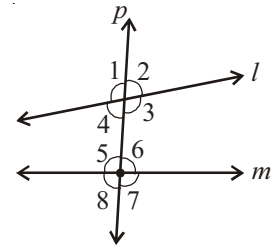
Try This

How many transversals can be drawn for two distinct lines?

4.2.1 Angles made by a transversal

When a transversal cuts two lines, 8 angles are formed. This is because at each intersection 4 angles are formed. Observe the figure.

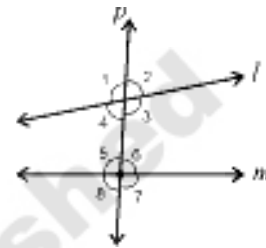
Here ' l ' and ' m ' are two lines intersected by the transversal ' p '. Eight angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7$ and $\angle 8$ are formed.



Angles $\angle 3, \angle 4, \angle 5$ and $\angle 6$, are lying inside ' l ' and ' m '.

They are thus called interior angles. The angles $\angle 1, \angle 2, \angle 7$ and $\angle 8$ are on the outside of the lines ' l ' and ' m '. They are thus called exterior angles.

Look at adjacent figure.



$\angle 1, \angle 2, \angle 7$ and $\angle 8$ are exterior angles.

$\angle 3, \angle 4, \angle 5$ and $\angle 6$ are interior angles.

We have learnt about vertically opposite angles and noted the fact that they are equal.

Renu looked at figure and said $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

Which are the other two pairs of vertically opposite angles?

She said that each exterior angle is paired with an vertically opposite angle which is in the interior. The angles in these pairs are equal. Do you agree with Renu?

Do This

- Identify the transversal in Figure (i) and (ii).

Identify the exterior and interior angles and fill the table given below:

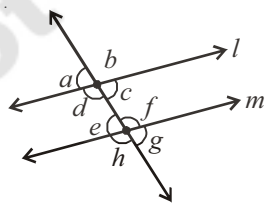


Figure (i)

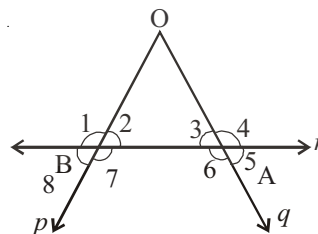
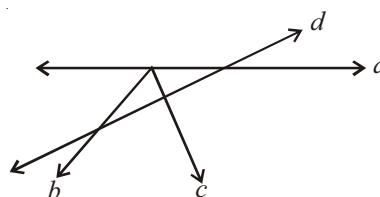
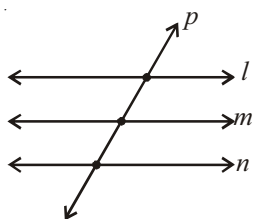


Figure (ii)

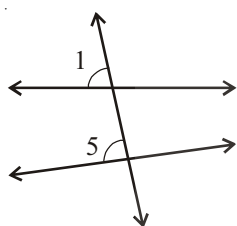
Figure	Transversal	Exterior angles	Interior angles
(i)			
(ii)			

2. Consider the following lines. Which line is a transversal? Number and list all the angles formed. Which are the exterior angles and which are the interior angles?

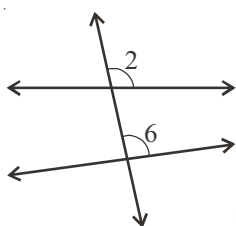


4.2.1 (a) Corresponding Angles

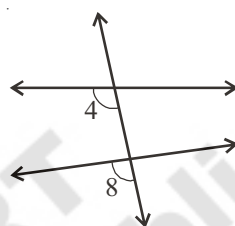
Look at figures (i), (ii), (iii) and (iv) below-



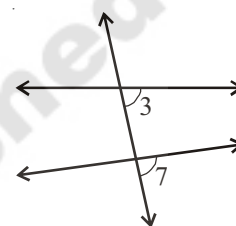
(i)



(ii)



(iii)

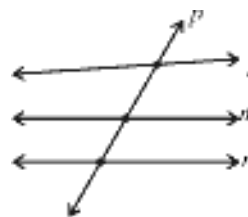


(iv)

Consider the pairs of angles $(\angle 1, \angle 5)$, $(\angle 2, \angle 6)$, $(\angle 4, \angle 8)$, $(\angle 3, \angle 7)$. Is there something common among these pairs of angles? These angles lie on different vertices. They are on the same side of the transversal and in each pair one is an interior angle and the other is an exterior angle.

Each of the above pair of angles is called corresponding angles.

What happens when a line is transversal to three lines? Which are the corresponding angles in this case? What is the number of exterior and interior angles in this case?



What would happen if number of lines intersected by the transversal becomes 4,5 and more?

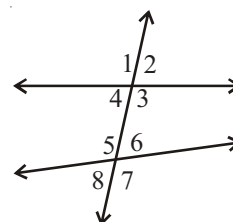
Can you predict the number of exterior and the interior angles that are corresponding to each other.

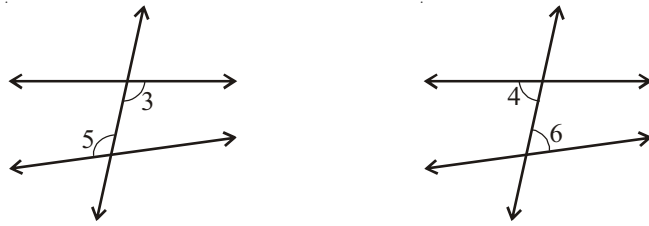
4.2.1 (b) Interior and Exterior Alternate Angles

Look at the adjacent figure. Find the angles which have the following three properties:

- (i) Have different vertices.
- (ii) Are on the either side of the transversal
- (iii) Lie 'between' the two lines (i.e are interior angles).

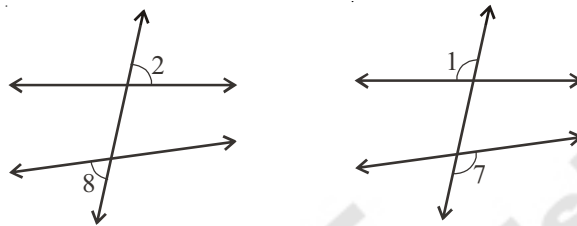
Such pairs of angles are called interior alternate angles.





The pairs of angles ($\angle 3, \angle 5$) and ($\angle 4, \angle 6$) are the two pairs of interior alternate angles.

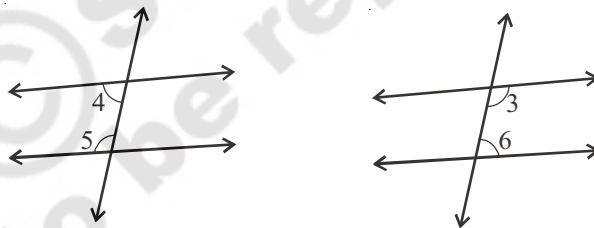
Similarly, you may find two pairs of exterior alternate angles.



The pairs of angles ($\angle 2, \angle 8$) and ($\angle 1, \angle 7$) are called alternate exterior angles.

4.2.1 (c) Interior Angles on the same side of the transversal

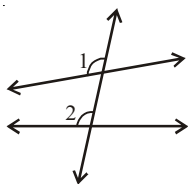
Interior angles can be on the same side of the transversal too.



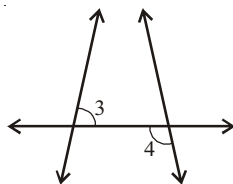
Angles $\angle 4, \angle 5$ and $\angle 3, \angle 6$ are the two pairs of interior angles on the same side of the transversal.

Do This

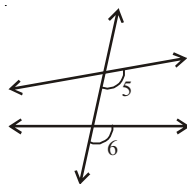
- Name the pairs of angles in each figure by their property.



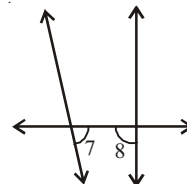
(i)



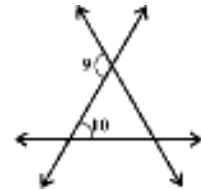
(ii)



(iii)



(iv)

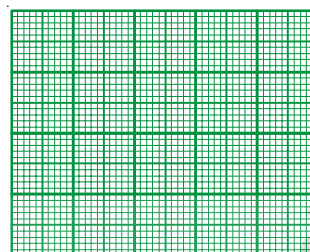
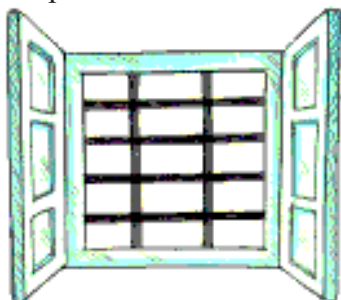


(v)

4.2.2 Transversal on parallel lines

You know that two coplanar lines which do not intersect are called parallel lines. Let us look at transversals on parallel lines and the properties of angles on them.

Look at the pictures of a window and a graph paper.



These give examples for parallel lines with a transversal.

Do This

Take a ruled sheet of paper. Draw two lines ' l ' and ' m ' parallel to each other.

Draw a transversal ' p ' on these lines.

Label the pairs of corresponding angles as shown in Figures (i), (ii), (iii) and (iv).

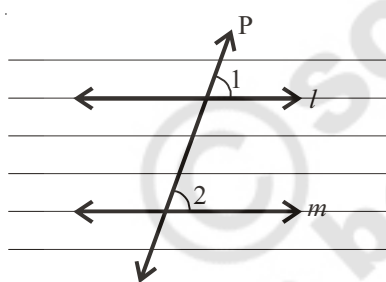


Figure (i)

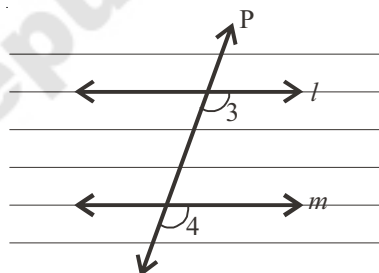


Figure (ii)

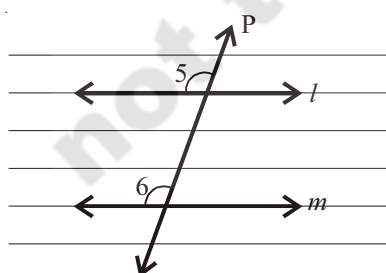


Figure (iii)

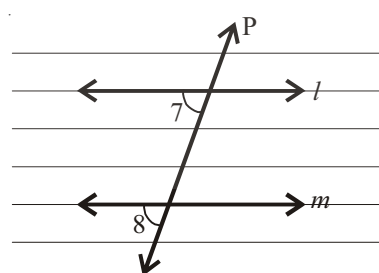


Figure (iv)

Place the tracing paper over Figure (i). Trace the lines ' l ', ' m ' and ' p '. Slide the tracing paper along ' p ', until the line ' l ' coincides with line ' m '. You find that $\angle 1$ on the traced figure coincides with $\angle 2$ of the original figure.

Thus $\angle 1 = \angle 2$

Are the remaining pairs of corresponding angles equal? Check by tracing and sliding.

You will find that if a pair of parallel lines are intersected by a transversal then the angles in each pair of corresponding angles are equal.

We can use this ‘corresponding angles’ property to get another result.

In the adjacent figure ‘ l ’ and ‘ m ’ are a pair of parallel lines and ‘ p ’ is a transversal.

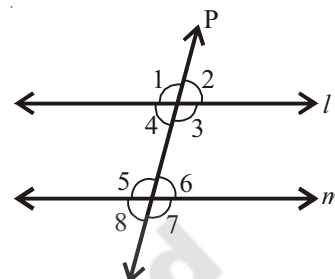
As all pairs of corresponding angles are equal,

$$\angle 1 = \angle 5$$

But $\angle 1 = \angle 3$ (vertically opposite angles)

Thus, $\angle 3 = \angle 5$

Similarly, you can show that $\angle 4 = \angle 6$.



Therefore, if a pair of parallel lines are intersected by a transversal then the angles in each pair of alternate interior angles are equal.

Do you find the same result for exterior alternate angles? Try.

Now, we find one more interesting result about interior angles on the same side of the transversal.

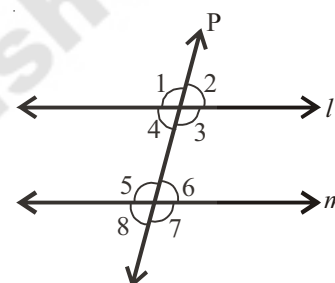
In the adjacent figure ‘ l ’ and ‘ m ’ a pair of parallel lines intersected by a transversal ‘ p ’.

$\angle 3 = \angle 5$ (alternate interior angles)

But $\angle 3 + \angle 4 = 180^\circ$ (Why?)

Thus, $\angle 4 + \angle 5 = 180^\circ$

Similarly $\angle 3 + \angle 6 = 180^\circ$ (Give reason)



Thus, if a pair of parallel lines are intersected by a transversal then the angles in each pair of interior angles on the same side of the transversal are supplementary.

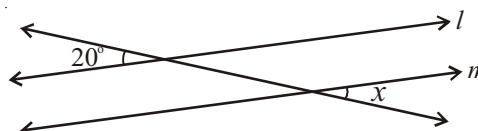
Example 1 : In the figure given below, ‘ l ’ and ‘ m ’ are a pair of parallel lines.

‘ p ’ is a transversal. Find ‘ x ’.

Solution : Given $l \parallel m$, p is a transversal.

$\angle x$ and 20° are a pair of exterior alternate angles, therefore are equal.

Thus, $\angle x = 20^\circ$.



Do This

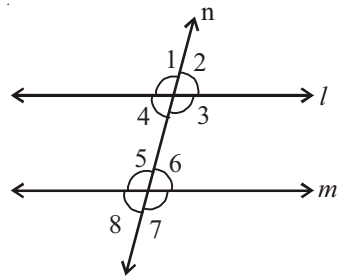


Figure (i)

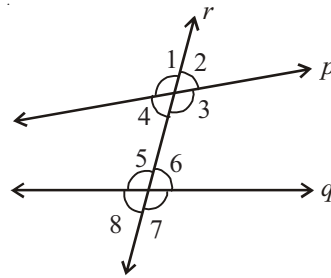


Figure (ii)

Trace the copy of figures (i) and (ii) in your note book. Measure the angles using a protractor and fill the table.

Table 1 : Fill the table with the measures of the corresponding angles.

Figure	Pairs of corresponding angles			
	1 st pair	2 nd pair	3 rd pair	4 th pair
(i)	$\angle 1 = \dots\dots\dots$	$\angle 2 = \dots\dots\dots$	$\angle 3 = \dots\dots\dots$	$\angle 4 = \dots\dots\dots$
	$\angle 5 = \dots\dots\dots$	$\angle 6 = \dots\dots\dots$	$\angle 7 = \dots\dots\dots$	$\angle 8 = \dots\dots\dots$
(ii)	$\angle 1 = \dots\dots\dots$	$\angle 2 = \dots\dots\dots$	$\angle 3 = \dots\dots\dots$	$\angle 4 = \dots\dots\dots$
	$\angle 5 = \dots\dots\dots$	$\angle 6 = \dots\dots\dots$	$\angle 7 = \dots\dots\dots$	$\angle 8 = \dots\dots\dots$

Find out in which figure the pairs of corresponding angles are equal?

What can you say about the lines 'l' and 'm' ?

What can you say about the lines 'p' and 'q' ?

Which pair of lines is parallel?

Thus, when a transversal intersects two lines and the pair of corresponding angles are equal then the lines are parallel.

Table 2 : Fill the table with the measures of the interior alternate angles.

Figure	Pairs of interior alternate angles	
	1 st pair	2 nd pair
(i)	$\angle 3 = \dots\dots\dots$ $\angle 5 = \dots\dots\dots$	$\angle 4 = \dots\dots\dots$ $\angle 6 = \dots\dots\dots$
(ii)	$\angle 3 = \dots\dots\dots$ $\angle 5 = \dots\dots\dots$	$\angle 4 = \dots\dots\dots$ $\angle 6 = \dots\dots\dots$

Find out in which figure the pairs of interior alternate angles are equal?

What can you say about the lines '*l*' and '*m*' ?

What can you say about the lines '*p*' and '*q*'?

Thus, if a pair of lines are intersected by a transversal and the alternate interior angles are equal then the lines are parallel.

Table 3 : Fill the table with the measures of interior angles on the same side of the transversal

Figure	Pairs of interior angles on the same side of the transversal.			
	1 st pair		2 nd pair	
(i)	$\angle 3 = \dots\dots\dots$ $\angle 6 = \dots\dots\dots$	$\angle 3 + \angle 6 = \dots\dots\dots$	$\angle 4 = \dots\dots\dots$ $\angle 5 = \dots\dots\dots$	$\angle 4 + \angle 5 = \dots\dots\dots$
(ii)	$\angle 3 = \dots\dots\dots$ $\angle 6 = \dots\dots\dots$	$\angle 3 + \angle 6 = \dots\dots\dots$	$\angle 4 = \dots\dots\dots$ $\angle 5 = \dots\dots\dots$	$\angle 4 + \angle 5 = \dots\dots\dots$

In which figure the pairs of interior angles on the same side of the transversal are supplementary (i.e sum is 180°)?

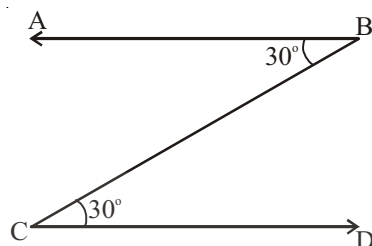
What can you say about the lines '*l*' and '*m*' ?

What can you say about the lines '*p*' and '*q*' ?

Thus, if a pair of lines are intersected by a transversal and the interior angles on the same side of the transversal are supplementary then the lines are parallel.

Example 2 : In the figure given below, two angles are marked as 30° each.

Is $AB \parallel CD$? How?



Solution : The given angles form a pair of interior alternate angles with transversal \overline{BC} .

As the angles are equal, $\overline{AB} \parallel \overline{CD}$.



Exercise - 7

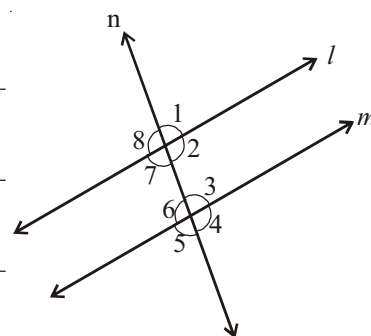
1. Fill up the blanks-

- (i) The line which intersects two or more lines at distinct points is called _____
- (ii) If the pair of alternate interior angles are equal then the lines are _____
- (iii) The sum of interior angles on the same side of the transversal are supplementary then the lines are _____
- (iv) If two lines intersect each other then the number of common points they have _____.

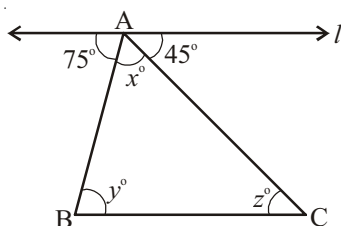
2. In the adjacent figure, the lines ' l ' and ' m ' are parallel and ' n ' is a transversal.

Fill in the blanks for all the situations given below-

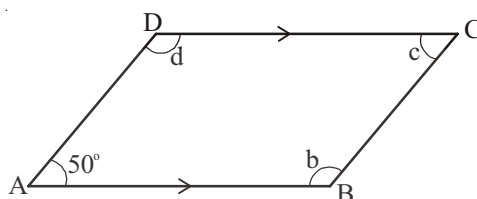
- (i) If $\angle 1 = 80^\circ$ then $\angle 2 =$ _____
- (ii) If $\angle 3 = 45^\circ$ then $\angle 7 =$ _____
- (iii) If $\angle 2 = 90^\circ$ then $\angle 8 =$ _____
- (iv) If $\angle 4 = 100^\circ$ then $\angle 8 =$ _____



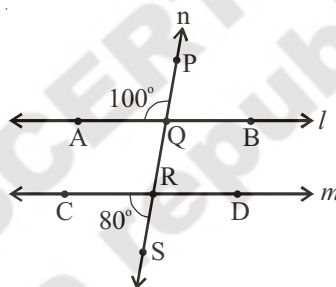
3. Find the measures of x, y and z in the figure, where $l \parallel BC$



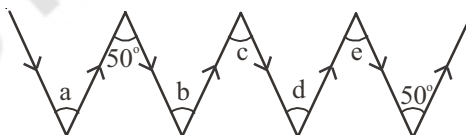
4. ABCD is a quadrilateral in which $AB \parallel DC$ and $AD \parallel BC$. Find $\angle b, \angle c$ and $\angle d$.



5. In a given figure, 'l' and 'm' are intersected by a transversal 'n'. Is $l \parallel m$?



6. Find $\angle a, \angle b, \angle c, \angle d$ and $\angle e$ in the figure? Give reasons.



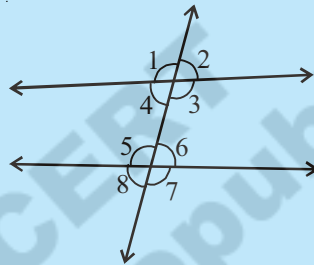
Note: Two arrow marks pointing in the same direction represent parallel lines.



Looking Back

- 1.(i) If the sum of two angles is equal to 90° , then the angles are called complementary angles.
 - (ii) Each angle in a pair of complementary angles is acute.
- 2.(i) If the sum of two angles is equal to 180° , then the angles are called supplementary angles.
 - (ii) Each angle in a pair of supplementary angles may be either acute or right or obtuse.
 - (iii) Two right angles always supplement to each other.

3. The angles formed on both sides of a common arm and a common vertex are adjacent angles.
4. A pair of complementary angles or a pair of supplementary angles need not be adjacent angles.
5. A pair of angles that are adjacent and supplementary form a linear pair.
- 6.(i) When two lines intersect each other at a point (vertex), the angles formed opposite to each other are called vertically opposite angles.
(ii) A pair of vertically opposite angles are always equal in measure
- 7.(i) A line which intersects two or more lines at distinct points is called a transversal to the lines.
(ii) A transversal makes eight angles with two lines as shown in the adjacent figure.



S.No.	Types of angles	No.of Pairs	Angles
1.	Interior angles	---	$\angle 3, \angle 4, \angle 5, \angle 6$
2.	Exterior angles	---	$\angle 1, \angle 2, \angle 7, \angle 8$
3.	Vertically opposite angles	4 pairs	$(\angle 1, \angle 3); (\angle 2, \angle 4); (\angle 5, \angle 7); (\angle 6, \angle 8)$
4.	Corresponding angles	4 pairs	$(\angle 1, \angle 5); (\angle 2, \angle 6); (\angle 4, \angle 8); (\angle 3, \angle 7)$
5.	Alternate interior angles	2 pairs	$(\angle 3, \angle 5); (\angle 4, \angle 6)$
6.	Alternate exterior angles	2 pairs	$(\angle 1, \angle 7); (\angle 2, \angle 8)$
7.	Interior angles on the same side of transversal	2 pairs	$(\angle 3, \angle 6); (\angle 4, \angle 5)$

8. When a transversal intersects a pair of parallel lines, the angles in
 - (i) Each pair of corresponding angles are equal.
 - (ii) Each pair of alternate interior angles are equal.
 - (iii) Each pair of alternate exterior angles are equal.
 - (iv) Each pair of interior angles on the same side of the transversal are supplementary.