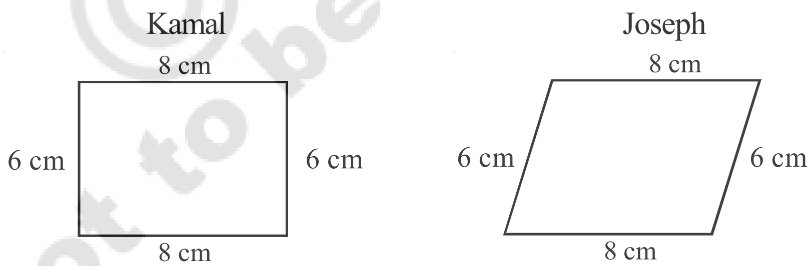
Chapter 3

Construction of Quadrilaterals

3.0 Introduction

We see fields, houses, bridges, railway tracks, school buildings, play grounds etc, around us. We also see kites, ludos, carrom boards, windows, blackboards and other things around. When we draw these things what do the figures look like? What is the basic geometrical shape in all these? Most of these are quadrilateral figures with four sides.

Kamal and Joseph are drawing a figure to make a frame of measurement with length 8 cm and breadth 6cm. They drew the their figures individually without looking at each other's figure.

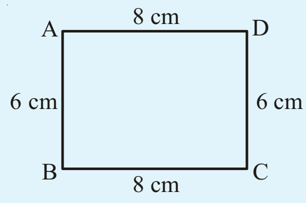


Are both the figures same?

You can see that both of these figures are quadrilaterals with the same measurements but the figures are not same. In class VII we have discussed about uniqueness of triangles. For a unique triangle you need any three measurements. They may be three sides or two sides and one included angle, two angles and a side etc. How many measurements do we need to make a unique quadrilateral? By a unique quadrilateral we mean that quadrilaterals made by different persons with the same measurements will be congruent.

Do This:

Take a pair of sticks of equal length, say 8 cm. Take another pair of sticks of equal length, say, 6 cm. Arrange them suitably to get a rectangle of length 8 cm and breadth 6 cm. This rectangle is created with the 4 available measurements. Now just push along the breadth of the rectangle. Does it still look alike? You will get a new shape of a rectangle Fig (ii), observe that the rectangle has now become a parallelogram. Have you altered the lengths of the sticks? No! The measurements of sides remain the same. Give another push to the newly obtained shape in the opposite direction; what do you get? You again get a parallelogram again, which is altogether different Fig (iii). Yet the four measurements remain the same. This shows that 4 measurements of a quadrilateral cannot determine its uniqueness. So, how many measurements determine a unique quadrilateral? Let us go back to the activity!



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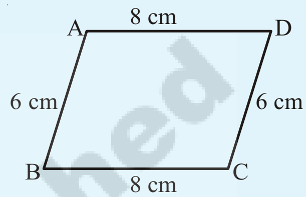

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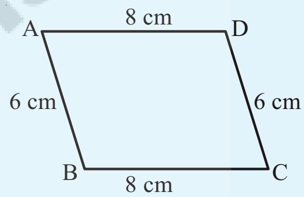
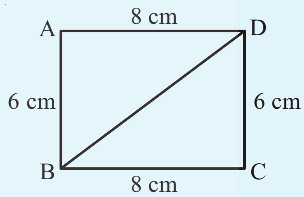


(iii)

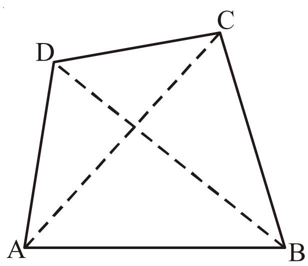
You have constructed a rectangle with two sticks each of length 8 cm and other two sticks each of length 6 cm. Now introduce another stick of length equal to BD and put it along BD (Fig iv). If you push the breadth now, does the shape change? No! It cannot, without making the figure open. The introduction of the fifth stick has fixed the rectangle uniquely, i.e., there is no other quadrilateral (with the given lengths of sides) possible now. Thus, we observe that five measurements can determine a quadrilateral uniquely. But will any five measurements (of sides and angles) be sufficient to draw a unique quadrilateral?



(iv)

3.1 Quadrilaterals and their Properties

In the Figure, ABCD is a quadrilateral. with vertices A, B, C, D and sides ; $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$. The angles of ABCD are $\angle ABC$, $\angle BCD$, $\angle CDA$ and $\angle DAB$ and the diagonals are $\overline{AC}$, $\overline{BD}$.



Do This

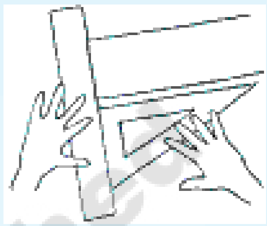
Equipment

You need: a ruler, a set square, a protractor.

Remember:

To check if the lines are parallel,

Slide set square from the first line to the second line as shown in adjacent figures.



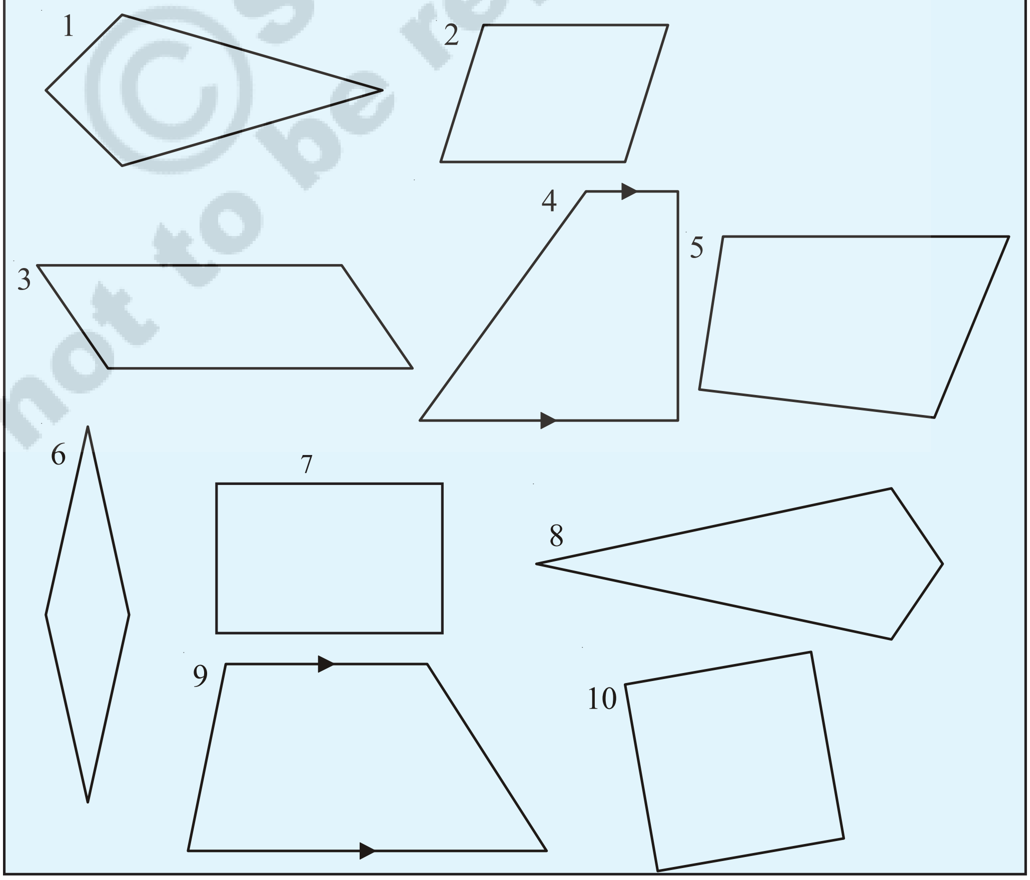
Now let us investigate the following using proper instruments.

For each quadrilateral.

(a) Check to see if opposite sides are parallel.

(b) Measure each angle.

(c) Measure the length of each side.



Record your observations and complete the table below.

Quadrilateral	2 pairs of parallel sides	1 pair of parallel sides	4 right angles	2 pairs of opposite sides equal	2 pairs of opposite angles equal	2 pairs of adjacent sides equal	4 sides equal
1	×	×	×	×	×	✓	×
2							
3							
4							
5							
6							
7							
8							
9							
10							

Parallelograms are quadrilaterals with 2 pairs of parallel sides.

(a) Which shapes are parallelograms?

(b) What other properties does a parallelogram have?

Rectangles are parallelograms with four right angles.

(a) Which shapes are rectangles?

(b) What properties does a rectangle have?

A rhombus is a parallelogram with four equal sides .

(a) Which could be called a rhombus?

(b) What properties does a rhombus have?

A square is a rhombus with four right angles .

(a) Which shapes are squares?

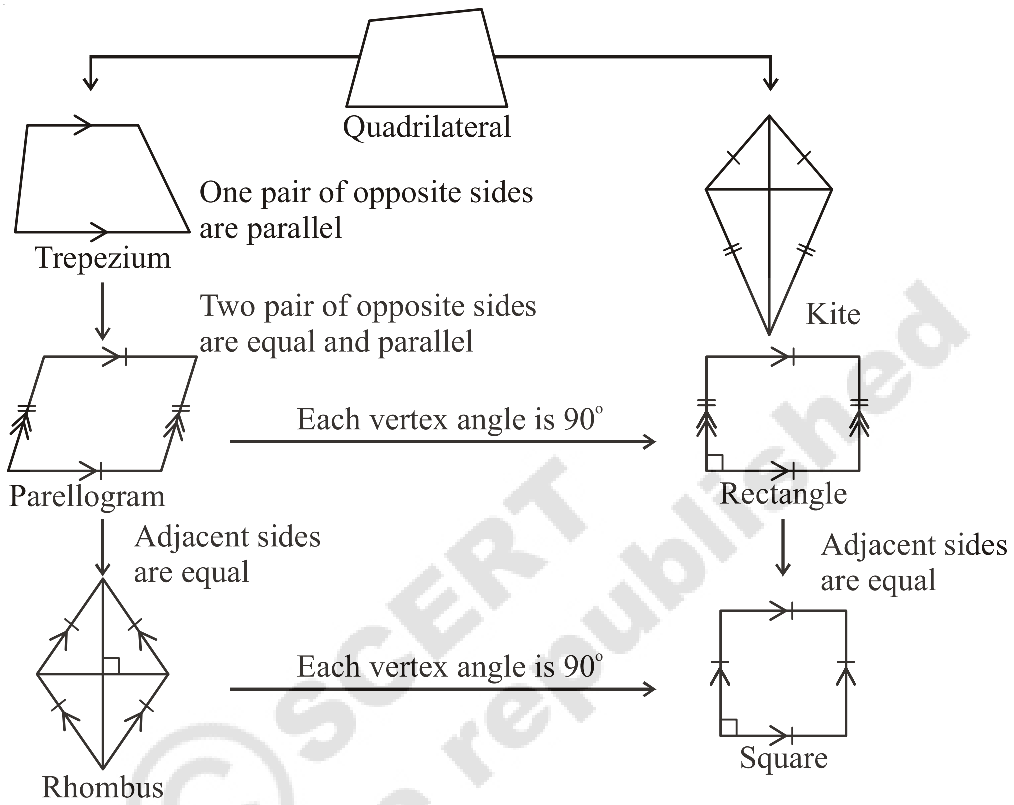
(b) What properties does a square have?

A trapezium is a quadrilateral with at least one pair of parallel sides.

(a) Which of the shapes could be called a trapezium and nothing else?

(b) What are the properties of a trapezium ?

Quadrilaterals 1 and 8 are **kites**. Write down some properties of kites.

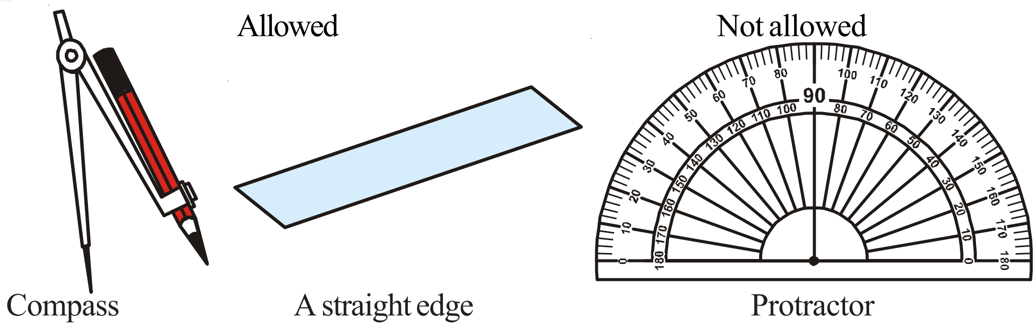


Think - Discuss and write :

1. Is every rectangle a paralellogram? Is every paralellogram a rectangle?
2. Uma has made a sweet chikki. She wanted it to be rectangular. In how many different ways can she verify that it is rectangular?

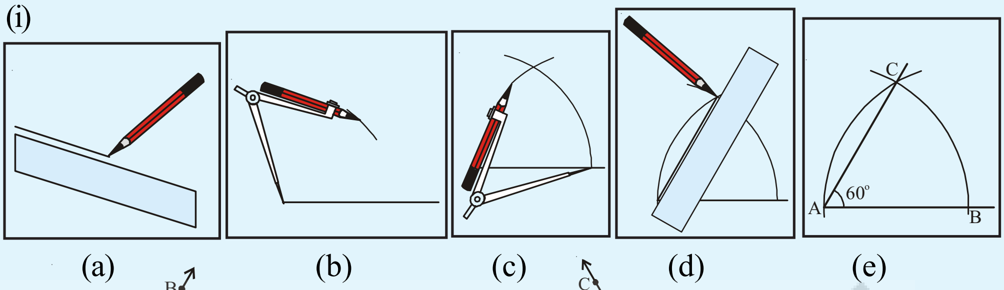
Do This

Can you draw the angle of 60^{o}



Observe the illustrations and write steps of construction for each.

(i)



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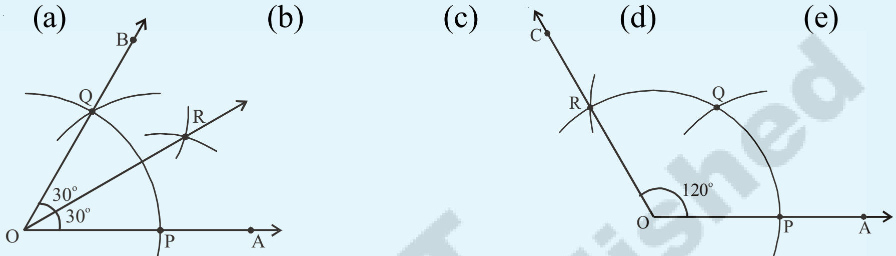
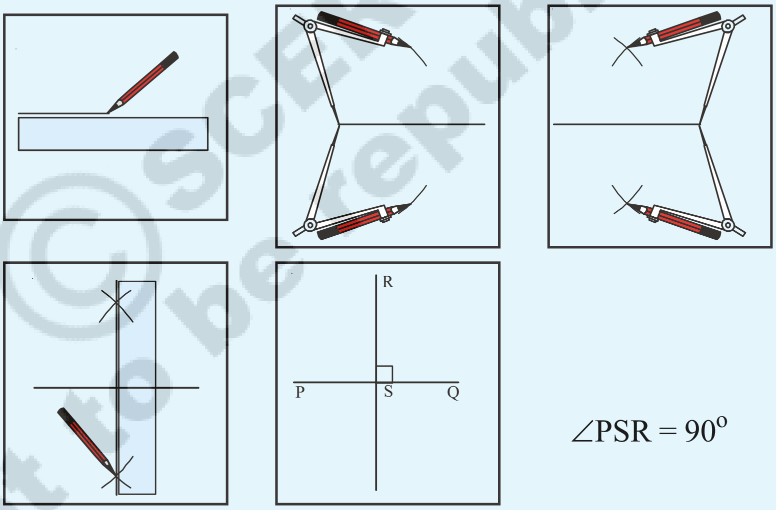


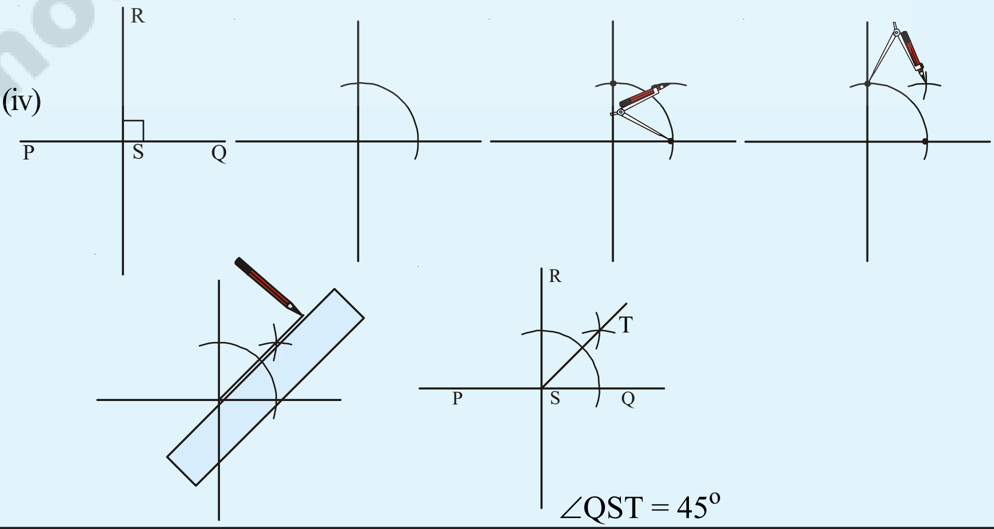
$\angle AOR = 30^o$ $\angle AOC = 120^o$

(iii)



$\angle PSR = 90^o$

(iv)



$\angle QST = 45^o$

3.2 Constructing a Quadrilateral

We would draw quadrilaterals when the following measurements are given.

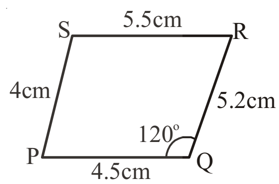
1. When four sides and one angle are given (S.S.S.S.A)
2. When four sides and one diagonal are given (S.S.S.S.D)
3. When three sides and two diagonals are given (S.S.S.D.D)
4. When two adjacent sides and three angles are given (S.A.S.A.A)
5. When three sides and two included angles are given (S.A.S.A.S)

3.2.1 Construction : When the lengths of four sides and one angle are given (S.S.S.S.A)

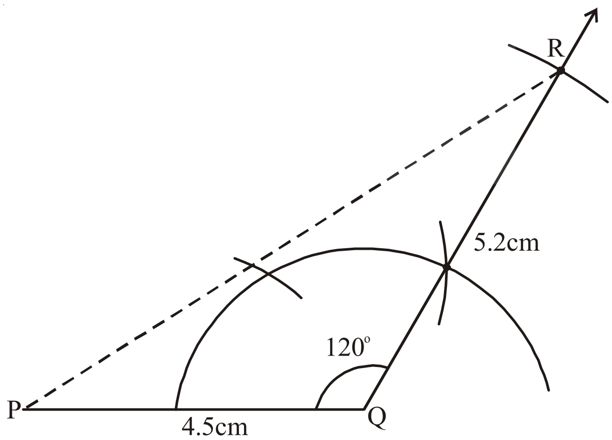
Example 1 : Construct a quadrilateral PQRS in which PQ = 4.5 cm, QR = 5.2 cm, RS = 5.5 cm, PS = 4 cm and $\angle$ PQR = 120^o.

Solution :

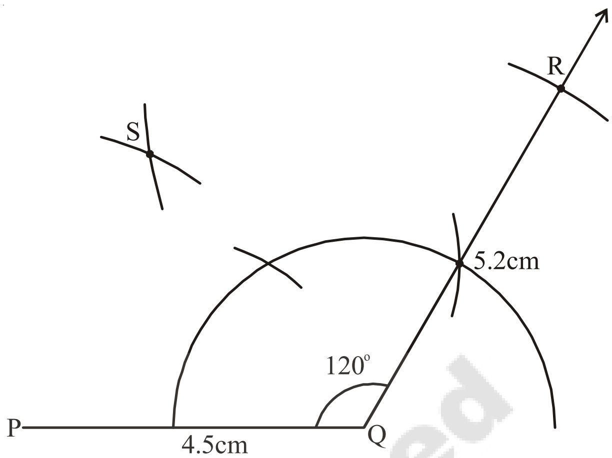
Step 1 : Draw a rough sketch of the required quadrilateral and mark the given measurements. Are they enough ?



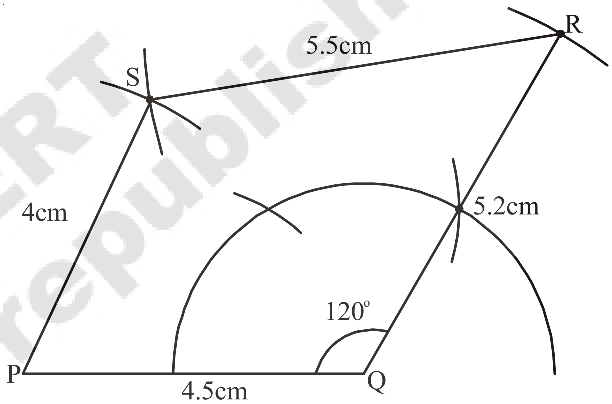
Step 2 : Draw ΔPQR using S.A.S Property of construction, by taking PQ = 4.5 cm, $\angle$PQR = 120^o and QR = 5.2 cm.



Step 3 : To locate the fourth vertex 'S', draw an arc, with centre P and radius 4cm (PS = 4 cm) Draw another arc with centre R and radius 5.5 cm (RS = 5.5 cm) which cuts the previous arc at S.



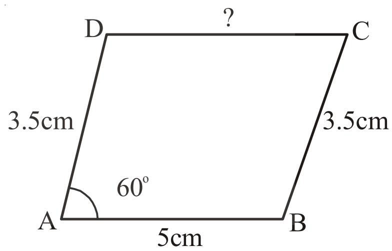
Step 4 : Join PS and RS to complete the required quadrilateral PQRS.



Example 2 : Construct parallelogram ABCD given that AB = 5 cm, BC = 3.5 cm and $\angle A = 60^{o}$.

Solution :

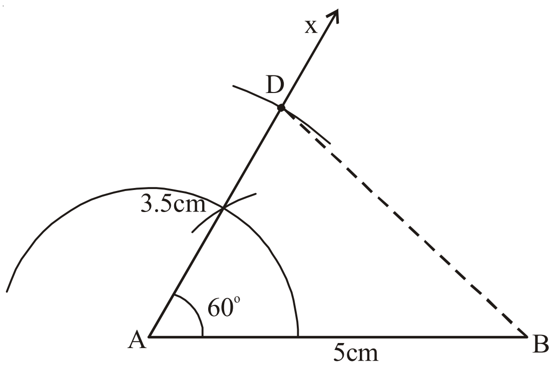
Step 1 : Draw a rough sketch of the parallelogram (a special type of quadrilateral) and mark the given measurements.



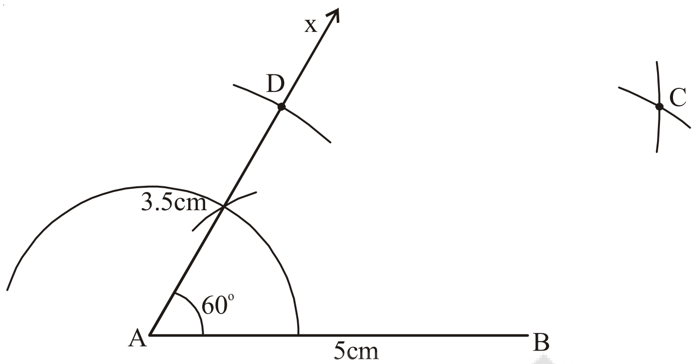
Here we are given only 3 measurements. But as the ABCD is a parallelogram we can also write that CD = AB = 5 cm and AD = BC = 3.5 cm. (How?)

(Now we got 5 measurements in total).

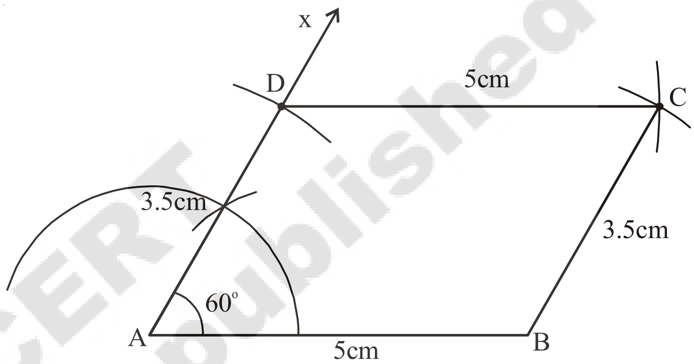
Steps 2: Draw ΔBAD using the measures AB = 5cm, $\angle A = 60^{o}$ and AD = 3.5 cm.



Steps 3: Locate the fourth vertex 'C' using other two measurements BC=3.5cm and DC = 5 cm.



Step 4 : Join B, C and C, D to complete the required parallelogram ABCD.



(Verify the property of the parallelogram using scale and protractor)

Let us generalize the steps of construction of quadrilateral.

Step 1: Draw a rough sketch of the figure .

Step 2 : If the given measurements are not enough, analyse the figure. Try to use special properties of the figure to obtain the required measurements

Step 3 : Draw a triangle with three of the five measurements and use the other measurements to locate the fourth vertex.

Step 4: Describe the steps of construction in detail.

Exercise - 3.1

Construct the quadrilaterals with the measurements given below :

(a) Quadrilateral ABCD with AB = 5.5 cm, BC = 3.5 cm, CD = 4 cm, AD = 5 cm and $\angle A = 45^{o}$.

(b) Quadrilateral BEST with BE = 2.9 cm, ES = 3.2 cm, ST = 2.7 cm, BT = 3.4 cm and $\angle B = 75^{o}$.

(c) Parallelogram PQRS with PQ = 4.5 cm, QR = 3 cm and $\angle PQR = 60^{o}$.

(d) Rhombus MATH with AT = 4 cm, $\angle MAT = 120^{o}$.

(e) Rectangle FLAT with FL = 5 cm, LA = 3 cm.

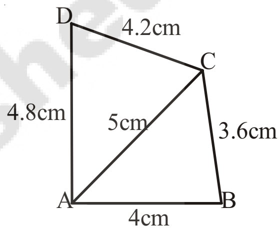
(f) Square LUDO with LU = 4.5 cm.

3.2.2 Construction : When the lengths of four sides and a diagonal is given (S.S.S.S.D)

Example 3 : Construct a quadrilateral ABCD where AB = 4 cm, BC = 3.6 cm, CD = 4.2 cm, AD = 4.8 cm and AC = 5 cm.

Solution :

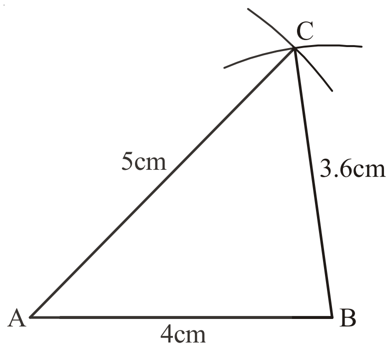
Step 1: Draw a rough sketch of the quadrilateral ABCD with the given data.



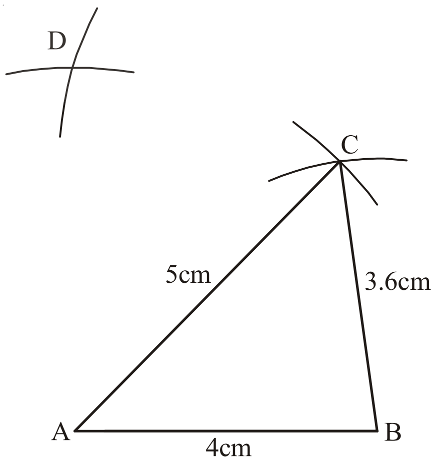
(Analyse if the given data is sufficient to draw the quadrilateral or not .

If sufficient then proceed further, if not conclude the that the data is not enough to draw the given figure).

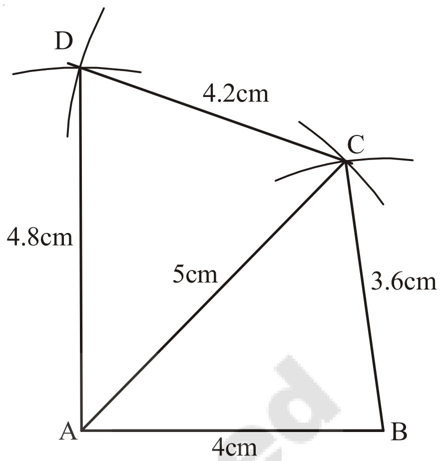
Step 2: Construct ΔABC with AB = 4 cm, BC = 3.6 cm and AC = 5 cm



Step 3: We have to locate the fourth vertex 'D'. It would be on the other side of AC. So with centre A and radius 4.8 cm (AD = 4.8 cm) draw an arc and with centre C and radius 4.2 cm (CD = 4.2 cm) draw another arc to cut the previous arc at D.



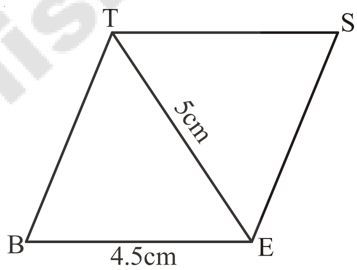
Step 4: Join A, D and C, D to complete the quadrilateral ABCD.



Example 4: Construct a rhombus BEST with BE = 4.5 cm and ET = 5 cm

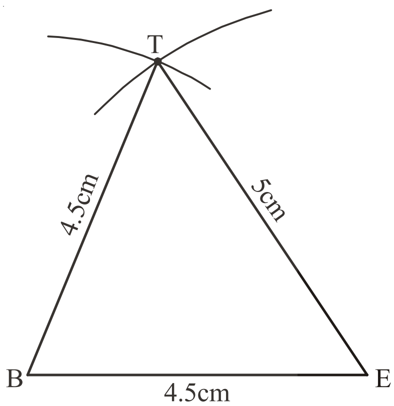
Solution :

Step 1 : Draw a rough sketch of the rhombus (a special type of quadrilateral). Hence all the sides are equal. So BE = ES = ST = BT = 4.5 cm and mark the given measurements.

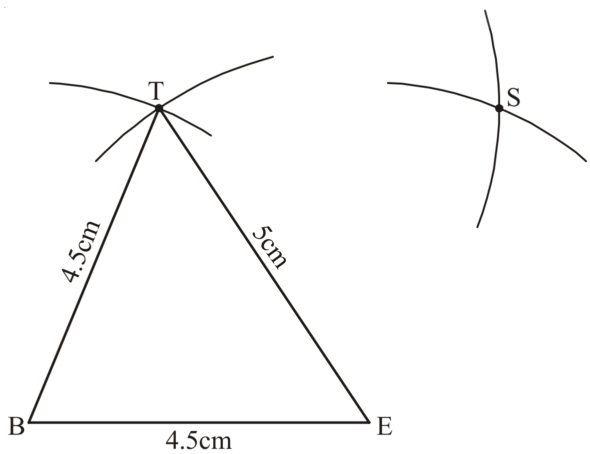


Now, with these measurements, we can construct the figure.

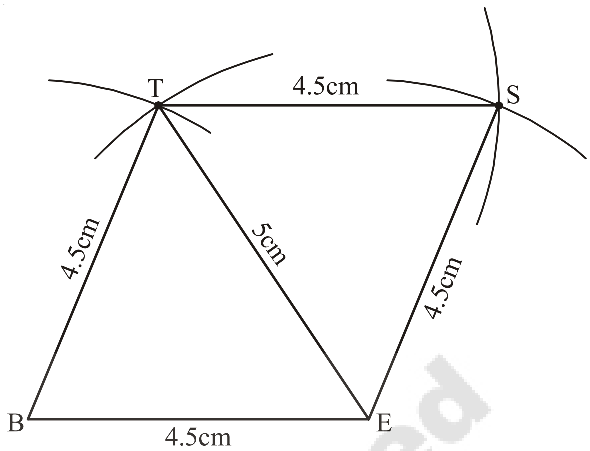
Step 2 : Draw ΔBET using SSS property of construction with measures BE = 4.5 cm, ET = 5 cm and BT = 4.5 cm



Step 3 : By drawing the arcs locate the fourth vertex 'S', with the remaining two measures ES = 4.5 cm and ST = 4.5 cm.



Step 4 : Join E, S and S, T to complete the required rhombus BEST.



Try These

1. Can you draw a parallelogram BATS where BA = 5 cm, AT = 6cm and AS = 6.5 cm ? explain?

2. A student attempted to draw a quadrilateral PLAY given that PL = 3 cm, LA = 4 cm, AY = 4.5 cm, PY = 2 cm and LY = 6 cm. But he was not able to draw it why ?

   Try to draw the quadrilateral yourself and give reason.

Exercise - 3.2

Construct quadrilateral with the measurements given below :

(a) Quadrilateral ABCD with AB = 4.5 cm, BC = 5.5 cm, CD = 4 cm, AD = 6 cm and AC = 7 cm

(b) Quadrilateral PQRS with PQ = 3.5 cm, QR = 4 cm, RS = 5 cm, PS = 4.5 cm and QS = 6.5 cm

(c) Parallelogram ABCD with AB = 6cm, CD = 4.5 cm and BD = 7.5 cm

(d) Rhombus NICE with NI = 4 cm and IE = 5.6 cm

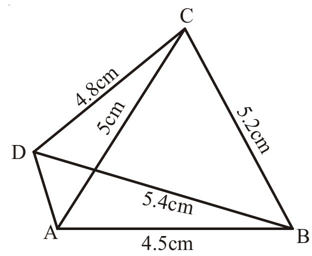
3.2.3 Construction: When the lengths of three sides and two diagonals are given (S.S.S.D.D)

Example 5 : Construct a quadrilateral ABCD, given that AB = 4.5 cm, BC = 5.2 cm, CD = 4.8 cm and diagonals AC = 5 cm and BD = 5.4 cm.

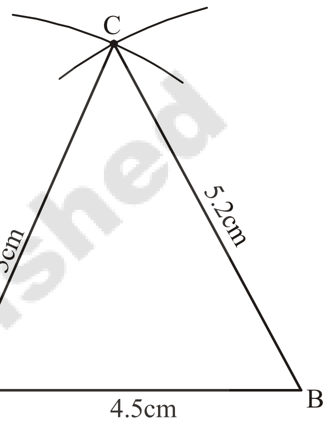
Solution :

Step 1: We first draw a rough sketch of the quadrilateral ABCD. Mark the given measurements.

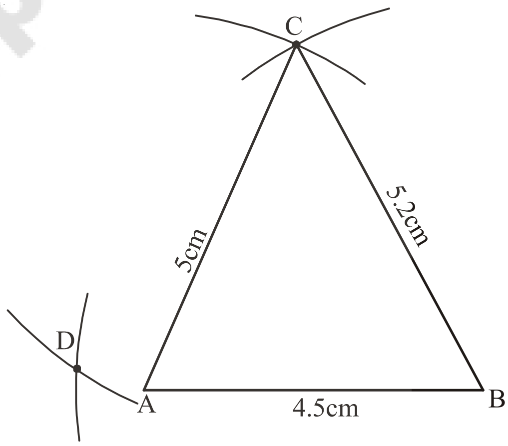
(It is possible to draw ΔABC with the available measurements)



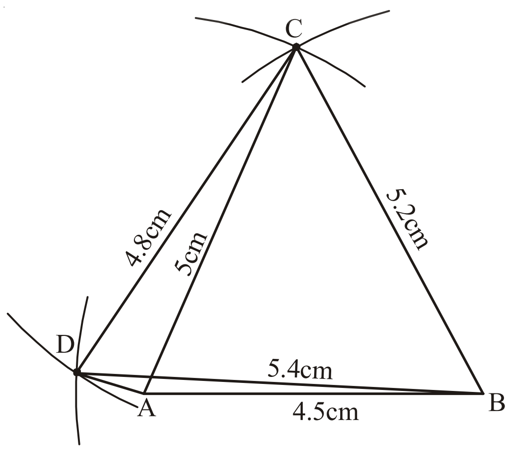
Step 2: Draw ΔABC using SSS Property of construction with measures AB = 4.5 cm, BC = 5.2 cm and AC = 5 cm



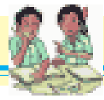
Step 3: With centre B and radius 5.4 cm and with centre C and radius 4.8 cm draw two arcs opposite to vertex B to locate D.



Step 4: Join C,D, B,D and A,D to complete the quadrilateral ABCD.



Think, Discuss and Write :



1. Can you draw the quadrilateral ABCD (given above) by constructing ΔABD first and then fourth vertex 'C' ? Give reason .
2. Construct a quadrilateral PQRS with PQ = 3 cm, RS = 3 cm, PS = 7.5 cm, PR = 8cm and SQ = 4 cm. Justify your result.

Exercise - 3.3

Construct the quadrilateral with the measurements given below :

(a) Quadrilateral GOLD OL = 7.5 cm, GL = 6 cm, LD = 5 cm, DG = 5.5 cm and OD = 10 cm

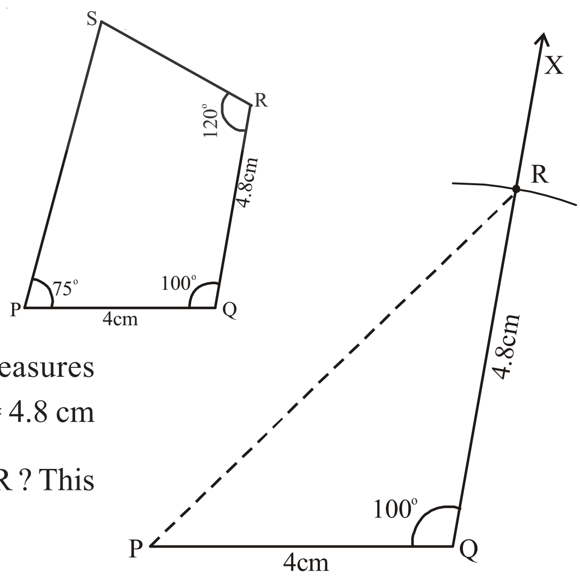
(b) Quadrilateral PQRS PQ = 4.2 cm, QR = 3 cm, PS = 2.8 cm, PR = 4.5 cm and QS = 5 cm.

3.2.4 Construction : When the lengths of two adjacent sides and three angles are known (S.A.S.A.A)

We construct the quadrilateral required as before but as many angles are involved in the construction use a ruler and a compass for standard angles and a protractor for others.

> The angles such as 0^o, 30^o, 45^o, 60^o, 90^o, 120^o and 180^o are called standard angles.

Example 6 : Construct a quadrilateral PQRS, given that PQ = 4 cm, QR = 4.8 cm, $\angle P = 75^o$ $\angle Q = 100^o$ and $\angle R = 120^o$.



Solution :

Step 1 : We draw a rough sketch of the quadrilateral and mark the given measurements. Select the proper instruments to construct angles.

Step 2: Construct ΔPQR using SAS property of construction with measures PQ = 4 cm, $\angle Q = 100^o$ and QR = 4.8 cm

(Why a dotted line is used to join PR ? This can be avoided in the next step).

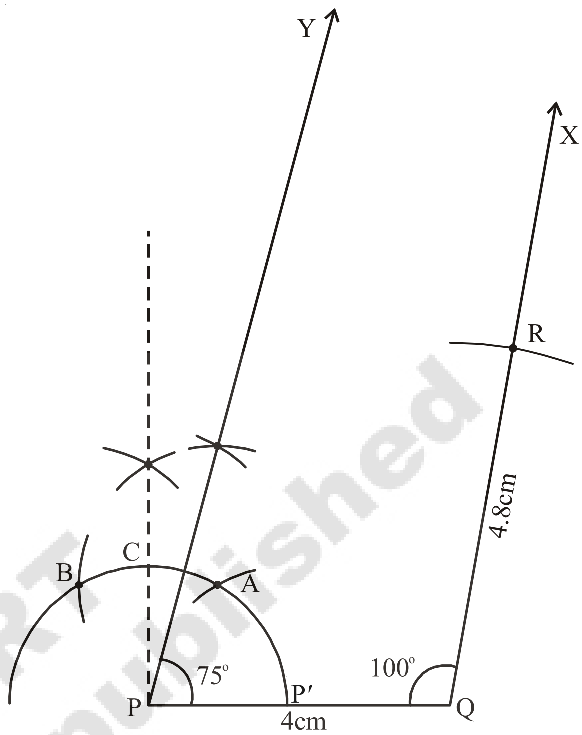
Step 3: Construct $\angle P = 75^{\circ}$ and draw $\overrightarrow{PY}$

[Do you understand how 75° is constructed?

(a) An arc is drawn from P. Let it intersect PQ at P'. With center P' and with the same radius draw two arcs to cut at two points A, B which give 60° and 120° respectively.

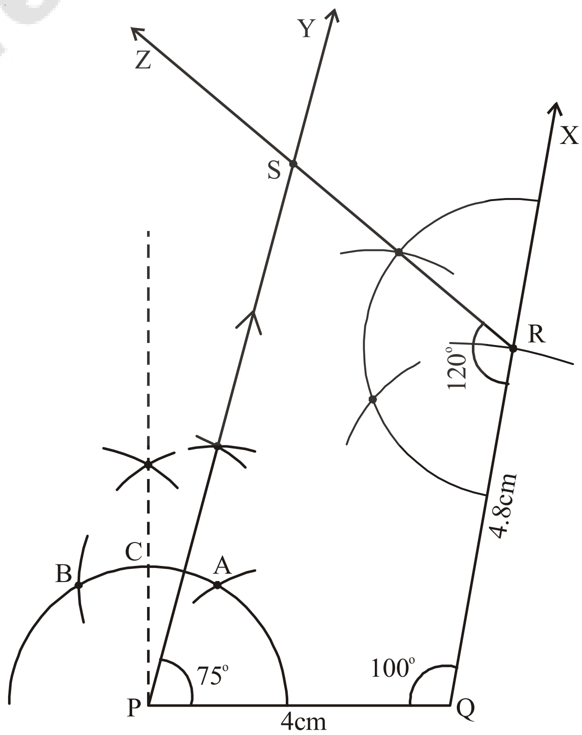
(b) From A,B construct an angular bisector. Which cuts the arc at C, making 90°.

(c) From A, C construct angular bisector (median of 60° and 90°) which is 75°.]

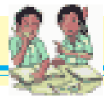


Step 4: Construct $\angle R = 120^{\circ}$ and draw $\overrightarrow{RZ}$ to meet $\overrightarrow{PY}$ at S.

PQRS is the required quadrilateral.



Think, Discuss and Write :



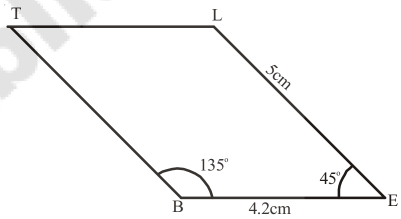
1. Can you construct the above quadrilateral PQRS, if we have an angle of 100^o at P instead of 75^o Give reason.
2. Can you construct the quadrilateral PLAN if PL = 6 cm, LA = 9.5 cm, $\angle P = 75^o$, $\angle L = 15^o$ and $\angle A = 140^o$.

   (Draw a rough sketch in each case and analyse the figure) State the reasons for your conclusion.

Example 7 : Construct a parallelogram BELT, given that BE = 4.2 cm, EL = 5 cm, $\angle T = 45^o$.

Solution :

Step 1: Draw a rough sketch of the parallelogram BELT and mark the given measurements. (Are they enough for construction ?)



Analysis :

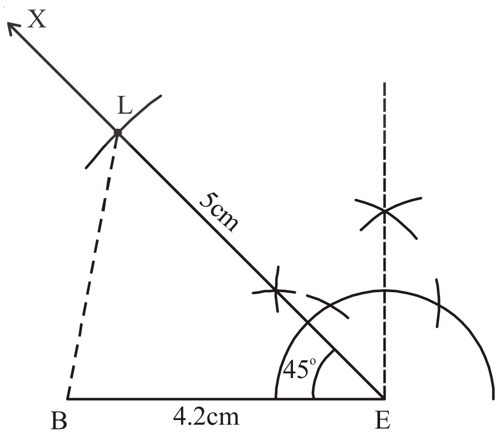
Since the given measures are not sufficient for construction, we shall find the required measurements using the properties of a parallelogram.

As "Opposite angles of a parallelogram are equal" so $\angle E = \angle T = 45^o$ and

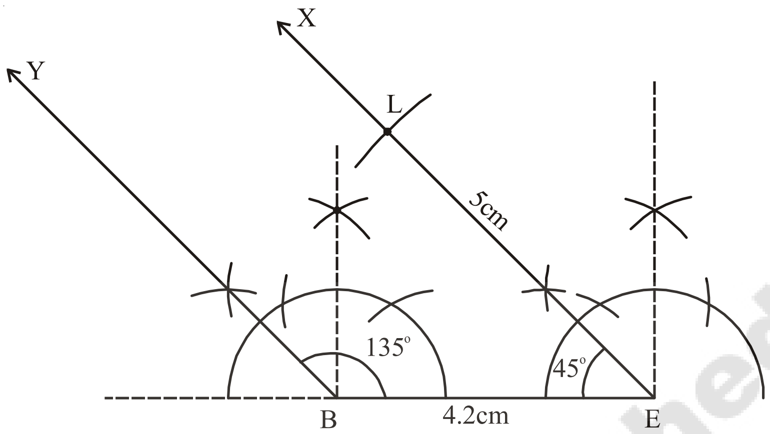
"The consecutive angles are supplementary" so $\angle L = 180^o - 45^o = 135^o$.

Thus $\angle B = \angle L = 135^o$

Step 2 : Construct ΔBEL using SAS property of construction model with BE = 4.2 cm, $\angle E = 45^o$ and EL = 5 cm

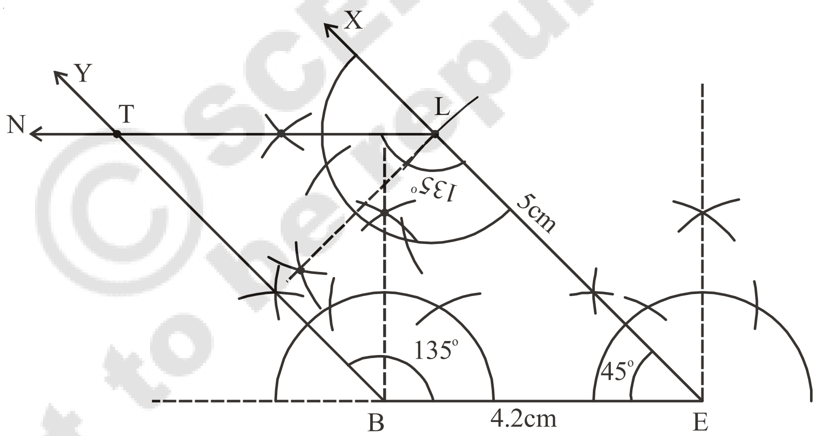


Step 3 : Construct $\angle B = 135^{o}$ and draw $\overrightarrow{BY}$



Step 4 : Construct $\angle L = 135^{o}$ and draw $\overrightarrow{LN}$ to meet $\overrightarrow{BY}$ at T.

BELT is the required quadrilateral (i.e. parallelogram)



Do This

Construct the above parallelogram BELT by using other properties of parallelogram?

Exercise - 3.4

Construct quadrilaterals with the measurements given below :

(a) Quadrilateral HELP with HE = 6cm, EL = 4.5 cm, $\angle H = 60^{o}$, $\angle E = 105^{o}$ and $\angle P = 120^{o}$.

(b) Parallelogram GRAM with GR = AM = 5 cm, RA = MG = 6.2 cm and $\angle R = 85^{o}$.

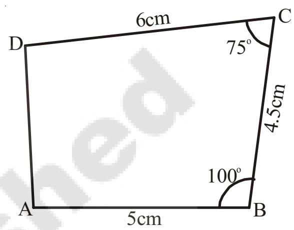
(c) Rectangle FLAG with sides FL = 6cm and LA = 4.2 cm.

3.2.5 Construction : When the lengths of three sides and two included angles are given (S.A.S.A.S)

We construct this type of quadrilateral by constructing a triangle with SAS property. Note particularly the included angles.

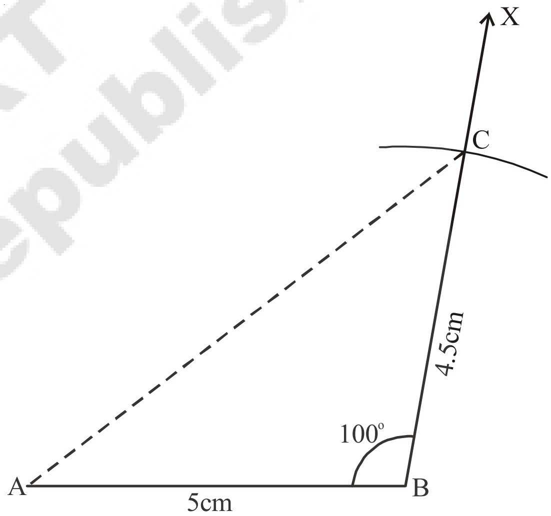
Example 8 : Construct a quadrilateral ABCD in which AB = 5cm, BC = 4.5 cm, CD = 6 cm, $\angle B = 100^o$ and $\angle C = 75^o$.

Solution :

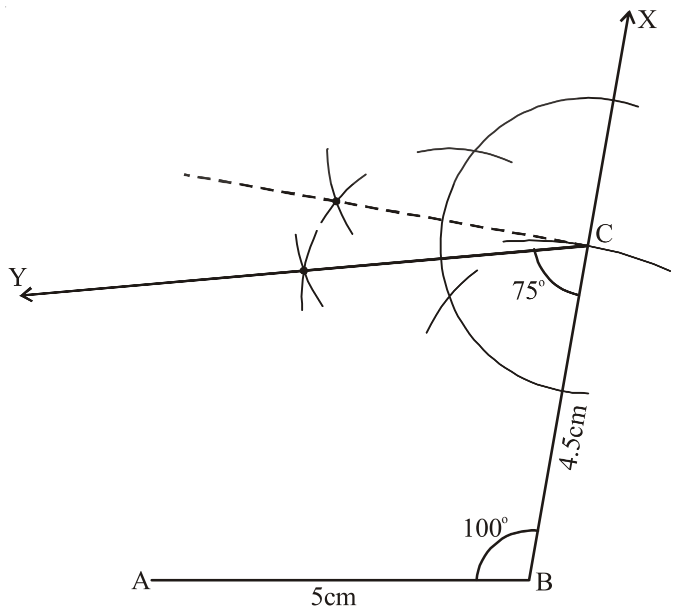


Step 1 : Draw a rough sketch, as usual and mark the measurements given (Find whether these measures are sufficient to construct a quadrilateral or not? If yes, proceed)

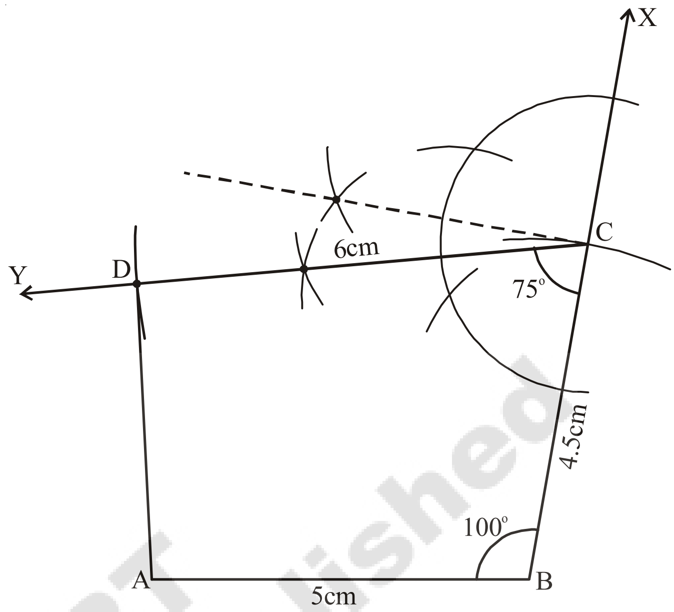
Step 2 : Draw ΔABC with measures AB= 5cm, $\angle B = 100^o$ and BC = 4.5 cm using SAS rule.



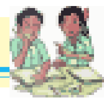
Step 3 : Construct $\angle C = 75^o$ and Draw $\overrightarrow{CY}$



Step 4 : With centre 'C' and radius 6 cm draw an arc to intersect $\overline{CY}$ at D. Join A, D. ABCD is the required quadrilateral.



Think, Discuss and Write :



Do you construct the above quadrilateral ABCD by taking BC as base instead of AB ? If So, draw a rough sketch and explain the various steps involved in the construction.

Exercise - 3.5

Construct following quadrilaterals-

(a) Quadrilateral PQRS with PQ $= 3.6$cm, QR $= 4.5$ cm, RS $= 5.6$ cm, $\angle PQR = 135^{o}$ and $\angle QRS = 60^{o}$.

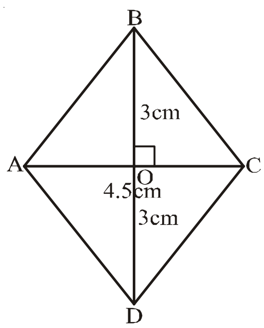
(b) Quadrilateral LAMP with AM $=$ MP $=$ PL $= 5$cm, $\angle M = 90^{o}$ and $\angle P = 60^{o}$.

(c) Trapezium ABCD in which AB $\|$ CD, AB $= 8$ cm, BC $= 6$cm, CD $= 4$cm and $\angle B = 60^{o}$.

3.2.6 Construction of Special types Quadrilaterals :

(a) Construction of a Rhombus :

Example 9 : Draw a rhombus ABCD in which diagonals AC $= 4.5$ cm and BD $= 6$ cm.

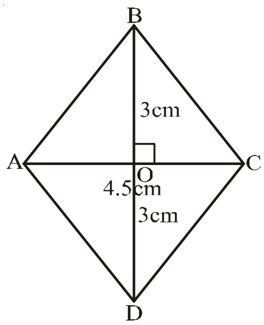


Solution :

Step 1 : Draw a rough sketch of rhombus ABCD and mark the given measurements. Are these measurements enough to construct the required figure ?

To examine this, we use one or other properties of rhombus to construct it.

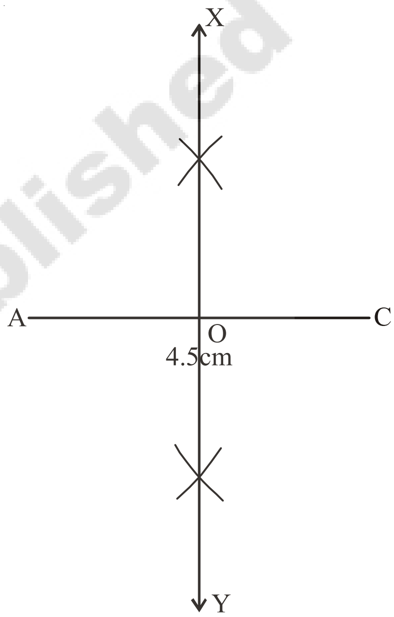
Analysis: The diagonals of a rhombus bisect each other perpendicularly, $\overline{AC}$ and $\overline{BD}$ are diagonals of the rhombus ABCD. Which bisect each other at 'O'. i.e. $\angle AOB = 90^{o}$ and



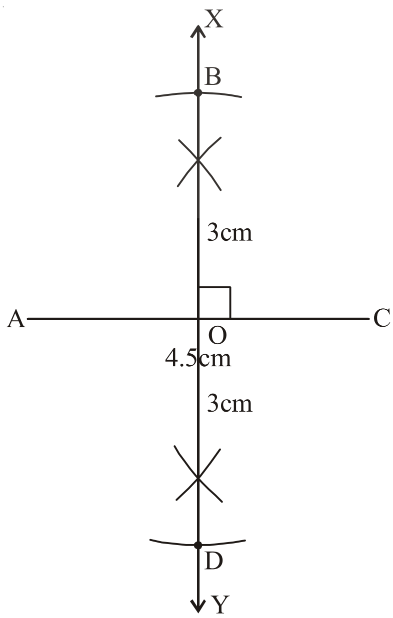
$$OB = OD = \frac{BD}{2} = \frac{6}{2} = 3\text{ cm}$$

Now proceed to step 2 for construction.

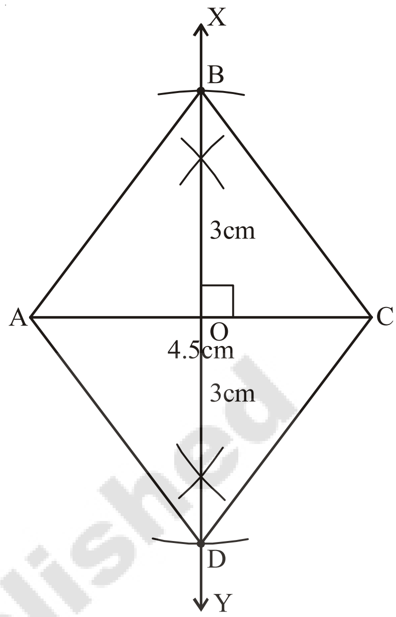
Step 2: Draw $\overline{AC} = 4.5$ cm (one diagonal of the rhombus ABCD) and draw a perpendicular bisector $\overline{XY}$ of it and mark the point of intersection as 'O'.



Step 3: As the other diagonal $\overline{BD}$ is Perpendicular to $\overline{AC}$, $\overline{BD}$ is a part of $\overline{XY}$. So with centre 'O' and radius 3 cm (OB = OD = 3cm) draw two arcs on either sides of $\overline{AC}$ to cut $\overline{XY}$ at B and D.



Step 4: Join A, B ; B, C ; C, D and D, A to complete the rhombus.



Think, Discuss and Write :

1. Can you construct the above quadrilateral (rhombus) taking BD as a base instead of AC? If not give reason.

2. Suppose the two diagonals of this rhombus are equal in length, what figure do you obtain? Draw a rough sketch for it. State reasons.

Exercise - 3.6

Construct quadrilaterals for measurements given below :

(a) A rhombus CART with CR = 6 cm, AT = 4.8 cm

(b) A rhombus SOAP with SA = 4.3 cm, OP = 5 cm

(c) A square JUMP with diagonal 4.2 cm.

What we have discussed

1. Five independent measurements are required to draw a unique quadrilateral
2. A quadrilateral can be constructed uniquely, if
   (a) The lengths of four sides and one angle are given
   (b) The lengths of four sides and one diagonal are given
   (c) The lengths of three sides and two diagonals are given
   (d) The lengths of two adjacent sides and three angles are given
   (e) The lengths of three sides and two included angles are given
3. The two special quadrilaterals, namely rhombus and square can be constructed when two diagonals are given.

Teachers Note:

Angles constructed by using compasses are accurate and can be proved logically, where as the protractor can be used for measurement and verification. So let our students learn to construct all possible angles with the help of compass.

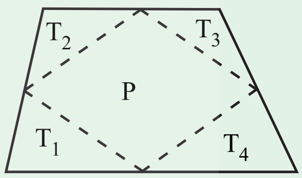
Fun with Paper Cutting

Tile and Smile

Cut a quadrilateral from a paper as shown in the figure. Locate the mid points of its sides, and then cut along the segments joining successive mid points to give four triangles T_1, T_2, T_3, T_4 and a parallelogram P.

Can you show that the four triangles tiles the parallelogram.

How does the area of the parallelogram compare to the area of the original quadrilateral.



Just for fun :

Qudrilateral + Quadrilateral = Parallelogram?

Fold a sheet of paper in half, and then use scissors to cut a pair of congruent convex quadrilaterals. Cut one of the quadrilateral along one of the diagonals, and the cut the second quadrilateral along the other diagonal. Show that four triangles can be arranged to form a parallelogram.