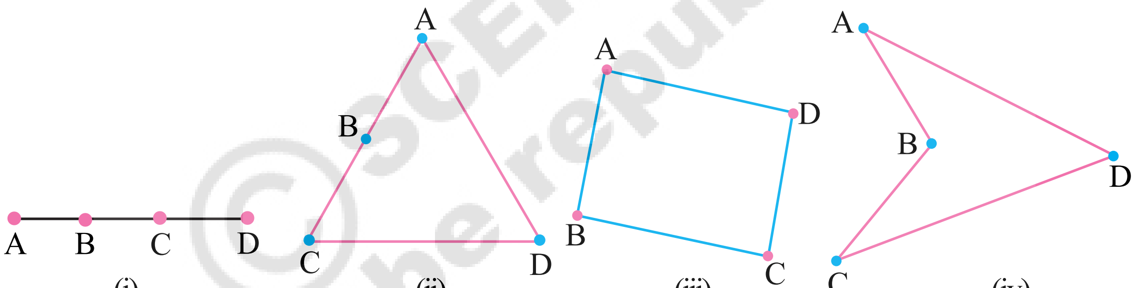
08 Quadrilaterals

8.1 Introduction

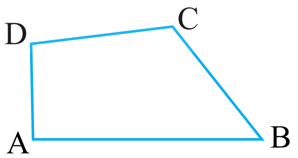
You have learnt many properties of triangles in the previous chapter with justification. You know that a triangle is a figure obtained by joining three non-collinear points in pairs. Do you know which figure you obtain with four points in a plane ? Note that if all the points are collinear, we obtain a line segment (Fig. (i)), if three out of four points are collinear, we get a triangle (Fig(ii)) and if any three points are not collinear, we obtain a closed figure with four sides (Fig (iii), (iv)), we call such a figure as a quadrilateral.



(i) (ii) (iii) (iv)

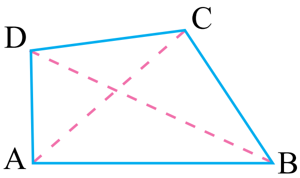
You can easily draw many more quadrilaterals and identify many around you. The Quadrilateral formed in Fig (iii) and (iv) are different in one important aspect. How are they different?

In this chapter we will study quadrilaterals only of type (Fig (iii)). These are convex quadrilaterals.



A quadrilateral is a simple closed figure bounded by four lines in a plane.

The quadrilateral ABCD has four sides AB, BC, CD and DA, four vertices are A, B, C and D. $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are the four angles formed at the vertices.



When we join the opposite vertices (A, C) and (B, D) (Fig (vi)) AC and BD are the two diagonals of the Quadrilateral ABCD.

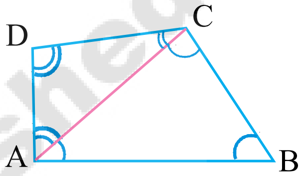
8.2 Properties of a Quadrilateral

There are four angles in the interior of a quadrilateral. Can we find the sum of these four angles? Let us recall the angle sum property of a triangle. We can use this property in finding sum of four interior angles of a quadrilateral.

ABCD is a quadrilateral and AC is a diagonal (see figure).

We know the sum of the three angles of ΔABC is,

$\angle CAB + \angle B + \angle BCA = 180^\circ$ ...(1) (Angle sum property of a triangle)



Similarly, in ΔADC,

$$\angle CAD + \angle D + \angle DCA = 180^\circ \quad ...(2)$$

Adding (1) and (2), we get

$$\angle CAB + \angle B + \angle BCA + \angle CAD + \angle D + \angle DCA = 180^\circ + 180^\circ$$

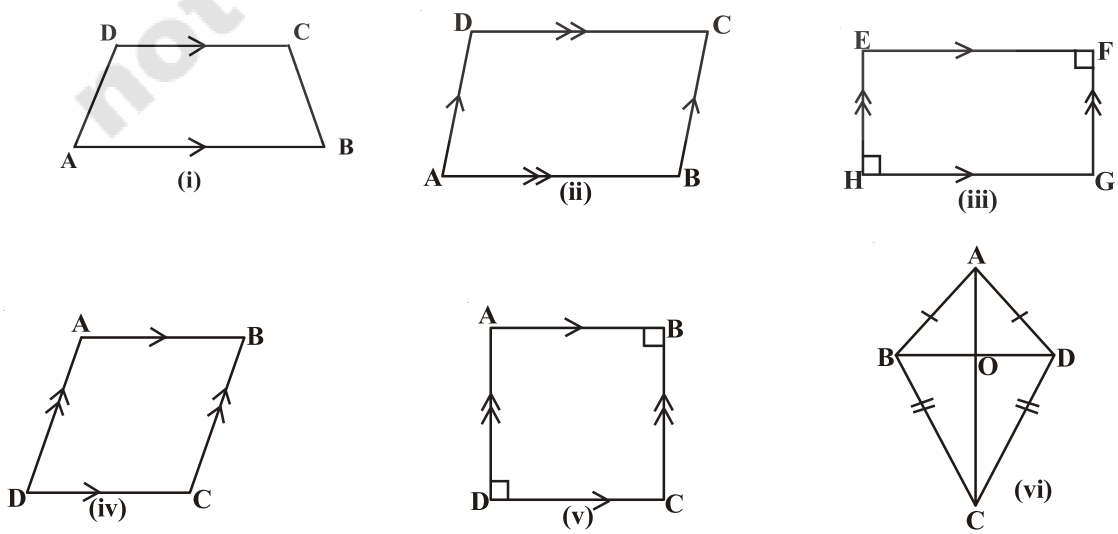
Since $\angle CAB + \angle CAD = \angle A$ and $\angle BCA + \angle DCA = \angle C$

So, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

i.e the sum of four angles of a quadrilateral is 360° or 4 right angles.

8.3 Different Types of Quadrilaterals

Look at the quadrilaterals drawn below. We have come across most of them earlier. We will quickly consider these and recall their specific names based on their properties.



We observe that

- In fig. (i) the quadrilateral ABCD had one pair of opposite sides AB and DC parallel to each other. Such a quadrilateral is called a trapezium.

  If in a trapezium non parallel sides are equal, then the trapezium is an isoceles trapezium.

- In fig. (ii) both pairs of opposite sides of the quadrilateral are parallel such a quadrilateral is called a parallelogram. Fig.(iii), (iv) and (v) are also parallelograms.
- In fig. (iii) parallelogram EFGH has all its angles as right angles. It is a rectangle.
- In fig. (iv) parallelogram has its adjacent sides equal and is called a Rhombus.
- In fig. (v) parallelogram has its adjacent sides equal and angles of 90° this is called a square.
- The quadrilateral ABCD in fig.(vi) has the two pairs of adjacent sides equal, i.e. AB = AD and BC = CD. It is called a kite.

Consider what Nisha says:

A rhombus can be a square or but all squares are not rhombuses.

Lalita Adds

All rectangles are parallelograms but all parallelograms are not rectangles.

Which of these statements you agree with?

Give reasons for your answer. Write other such statements about different types of quadrilaterals.

Illustrative examples

Example-1. ABCD is a parallelogram and $\angle A = 60^o$. Find the remaining angles.

Solution : The opposite angles of a parallelogram are equal.

So in a parallelogram ABCD

$\angle C = \angle A = 60^o$ and $\angle B = \angle D$

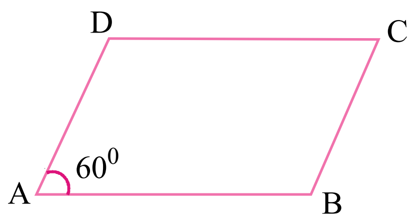
and the sum of consecutive angles of parallelogram is equal to 180^o.

As $\angle A$ and $\angle B$ are consecutive angles

$\angle D = \angle B = 180^o - \angle A$

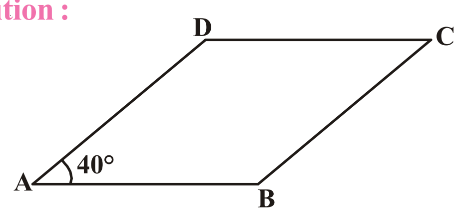
$= 180^o - 60^o = 120^o$.

Thus the remaining angles are 120^o, 60^o, 120^o.



Example-2. In a parallelogram ABCD, $\angle DAB = 40^o$ find the other angles of the parallelogram.

Solution :



ABCD is a parallelogram

$\angle DAB = \angle BCD = 40°$ and AD || BC

As sum of consective angles

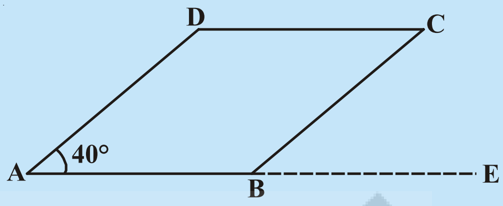
$\angle CBA + \angle DAB = 180°$

$\therefore \quad \angle CBA = 180 - 40°$

$= 140°$

Find this we can find $\angle ADC = 140°$ and $\angle BCD = 40°$

TRY THIS



Extend AB to E. Find $\angle CBE$. What do you notice. What kind of angles are $\angle ABC$ and $\angle CBE$?

Example-3. Two adjacent sides of a parallelogram are 4.5 cm and 3 cm. Find its perimeter.

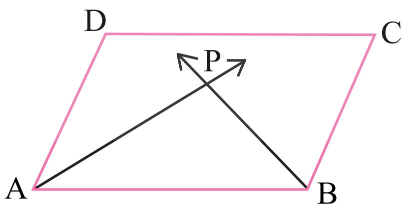
Solution : Since the opposite sides of a parallelogram are equal the other two sides are 4.5 cm and 3 cm.

Hence, the perimeter = 4.5 + 3 + 4.5 + 3 = 15 cm.

Example-4. In a parallelogram ABCD, the bisectors of the consecutive angles $\angle A$ and $\angle B$ intersect at P. Show that $\angle APB = 90^o$.

Solution : ABCD is a parallelogram $\overrightarrow{AP}$ and $\overrightarrow{BP}$ are bisectors of consecutive angles, $\angle A$ and $\angle B$.

As, the sum of consecutive angles of a parallelogram is supplementary,



$\angle A + \angle B = 180^o$

$\frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{180}{2}$

$\Rightarrow \angle PAB + \angle PBA = 90^o$

In Δ APB,

$\angle PAB + \angle APB + \angle PBA = 180^o$ (angle sum property of triangle)

$\angle APB = 180^0 - (\angle PAB + \angle PBA)$

$= 180^0 - 90^0$

$= 90^0$

Hence proved.

Exercise - 8.1

1. State whether the statements are True or False.
   - (i) Every parallelogram is a trapezium ( )
   - (ii) All parallelograms are quadrilaterals ( )
   - (iii) All trapeziums are parallelograms ( )
   - (iv) A square is a rhombus ( )
   - (v) Every rhombus is a square ( )
   - (vi) All parallelograms are rectangles ( )

2. Complete the following table by writing (YES) if the property holds for the particular Quadrilateral and (NO) if property does not holds.

	Properties	Trapezium	Parallelogram	Rhombus	Rectangle	square
a.	One pair of opposite sides are parallel	YES				
b.	Two pairs of opposite sides are parallel					
c.	Opposite sides are equal					
d.	Opposite angles are equal					
e.	Consecutive angles are supplementary					
f.	Diagonals bisect each other					
g.	Diagonals are equal					
h.	All sides are equal					
i.	Each angle is a right angle					
j.	Diagonals are perpendicular to each other.					

3. ABCD is trapezium in which AB || CD. If AD = BC, show that $\angle A = \angle B$ and $\angle C = \angle D$.

4. The four angles of a quadrilateral are in the ratio 1: 2:3:4. Find the measure of each angle of the quadrilateral.

5. ABCD is a rectangle AC is diagonal. Find the angles of ΔACD. Give reasons.

8.4 PARALLELOGRAM AND THEIR PROPERTIES

We have seen parallelograms are quadrilaterals. In the following we would consider the properties of parallelograms.

DO THIS

Cut-out a parallelogram from a sheet of paper again and cut along one of its diagonal. What kind of shapes you obtain? What can you say about these triangles?

Place one triangle over the other. Can you place each side over the other exactly. You may need to turn the triangle around to match sides. Since, the two traingles match exactly they are congruent to each other.

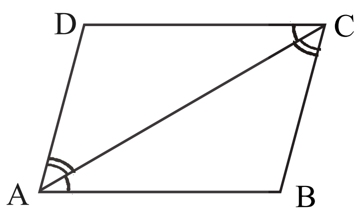
Do this with some more parallelograms. You can select any diagonal to cut along.

We see that each diagonal divides the parallelogram into two congruent triangles.

Let us now prove this result.

Theorem-8.1 : A diagonal of a parallelogram divides it into two congruent triangles.

Proof: Consider the parallelogram ABCD.



Join A and C. AC is a diagonal of the parallelogram.

Since AB || DC and AC is transversal

$\angle DCA = \angle CAB$. (Interior alternate angles)

Similarly DA || CB and AC is a transversal therefore $\angle DAC = \angle BCA$.

We have in ΔACD and ΔCAB

$\angle DCA = \angle CAB$ and $\angle DAC = \angle BCA$

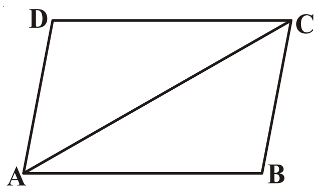
also $AC = CA$. (Common side)

Therefore $\Delta ABC \cong \Delta CDA$.

This means by the two traingles by A.S.A. rule (angle, side and angle) are congruent. This means that diagonal AC divides the parallelogram in two congruent parts.

Theorem-8.2 : In a parallelogram, opposite sides are equal.

Proof: We have already proved that a diagonal of a parallelogram divides it into two congruent triangles.



Thus in figure $\Delta ACD \cong \Delta CAB$

We have therefore $AB = DC$ and $\angle CBA = \angle ADC$

also $AD = BC$ and $\angle DAC = \angle ACB$

$\angle CAB = \angle DCA$

$\therefore \angle ACB + \angle DCA = \angle DAC + \angle CAB$

i.e. $\angle DAB = \angle DCB$

We thus have in a parallelogram

i. The opposite sides are equal.

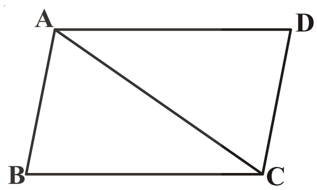
ii. The opposite angles are equal.

It can be noted that with opposite sides of a convex quadrilateral being parallel we can show the opposite sides and opposite angles are equal.

We will now try to show if we can prove the converse i.e. if the opposite sides of a quadrilateral are equal, then it is a parallelogram.

Theorem-8.3 : If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.

Proof : Consider the quadrilateral ABCD with $AB = DC$ and $BC = AD$.



Draw a diagonal AC.

Consider ΔABC and ΔCDA

We have $BC = AD$, $AB = DC$ and $AC = CA$ (Common side)

So $\Delta ABC \cong \Delta CDA$

Therefore $\angle BCA = \angle DAC$ with AC as transversal

or $AB \parallel DC$ ...(1)

Since $\angle ACD = \angle CAB$ with CA as transversal

We have $BC \parallel AD$ ...(2)

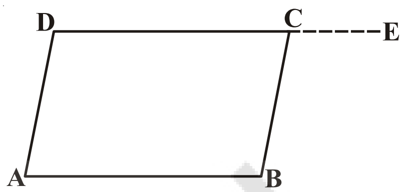
Therefore, ABCD is a parallelogram. By (1) and (2)

You have just seen that in a parallelogram both pairs of opposite sides are equal and conversely if both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram.

Can we show the same for a quadrilateral for which the pairs of opposite angles are equal?

Theorem-8.4 : In a quadrilateral, if each pair of opposite angles are equal then it is a parallelogram.

Proof: In a quadrilateral ABCD, $\angle A = \angle C$ and $\angle B = \angle D$ then prove that ABCD is a parallelogram.



We know $\angle A + \angle B + \angle C + \angle D = 360°$

$$\angle A + \angle B = \angle C + \angle D = \frac{360°}{2}$$

i.e. $\angle A + \angle B = 180°$

Extend DC to E

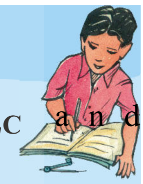
$\angle C + \angle BCE = 180°$ hence $\angle BCE = \angle ADC$

If $\angle BCE = \angle D$ then AD || BC (Why?)

With DC as a transversal

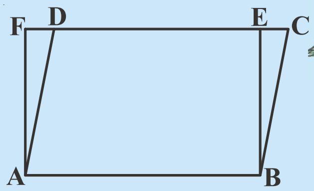
We can similarly show AB || DC or ABCD is a parallelogram.

EXERCISE - 8.2



1. In the adjacent figure ABCD is a parallelogram ABEF is a rectangle show that $\Delta AFD \cong \Delta BEC$.
2. Show that the diagonals of a rhombus divide it into four congruent triangles.
3. In a quadrilateral ABCD, the bisector of $\angle C$ and $\angle D$ intersect at O.

   Prove that $\angle COD = \frac{1}{2}(\angle A + \angle B)$



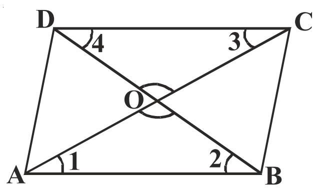
8.5 DIAGONALS OF A PARALLELOGRAM

Theorem-8.5 : The diagonals of a parallelogram bisect each other.

Proof: Draw a parallelogram ABCD.

Draw both of its diagonals AC and BD to intersect at the point 'O'.

In ΔOCD and ΔOAB



Mark the angles formed as $\angle 1, \angle 2, \angle 3, \angle 4$

$\angle 1 = \angle 3$ (AB || CD and AC transversal)

$\angle 2 = \angle 4$ (Why) (Interior alternate angles) and AB = CD (Property of parallelogram)

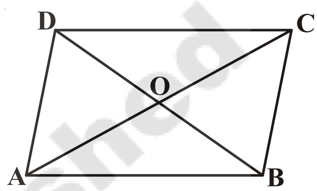
By A.S.A congruency property

$\Delta OCD \cong \Delta OAB$

CO= OA, DO = OB or diagonals bisect each other.

Hence we have to check if the converse is also true. Converse is if diagonals of a quadrilateral bisect each other then it is a parallelogram.

Theorem-8.6 : If the diagonals of a quadrilateral bisect each other then it is a parallelogram.



Proof: ABCD is a quadrilateral.

AC and BD are the diagonals intersect at 'O',

such that OA = OC and OB = OD.

Prove that ABCD is a parallelogram.

(**Hint** : Consider ΔAOB and ΔCOD. Are these congruent? If so then what can we say?)

8.5.1 More geometrical statemenets

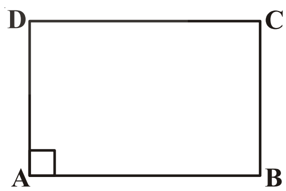
In the previous examples we have showed that starting from some general premises we can find many statements that we can make about a particular figure(Parallelogram). We use previous results to deduce new statements. Note that these statements need not be verified by measurements as they have been shown as true in all cases.

Such statements that are deduced from the previously known and proved statements are called corollary. A corollary is a statement the truth of which follows readily from an established theorem.

Corollary-1 : Show that each angle of a rectangle is a right angle.

Solution : Rectangle is a parallelogram in which one angle is a right angle.

We are given: ABCD is a rectangle. Let one angle is $\angle A = 90^o$



We have to show that $\angle B = \angle C = \angle D = 90^o$

Proof : Since ABCD is a parallelogram,

thus AD || BC and AB is a transversal

so $\angle A + \angle B = 180^o$ (Interior angles on the same side of a transversal)

as $\angle A = 90^o$ (given)

$\therefore \angle B = 180^o - \angle A$

$= 180^o - 90^o = 90^o$

Now $\angle C = \angle A$ and $\angle D = \angle B$ (opposite angles of parallelogram)

So $\angle C = 90^o$ and $\angle D = 90^o$.

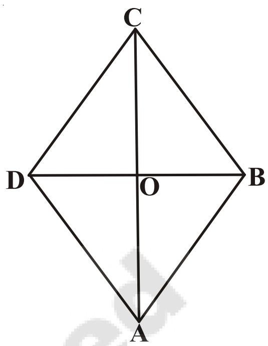
Therefore each angle of a rectangle is a right angle.

Corollary-2 : Show that the diagonals of a rhombus are perpendicular to each other.

Proof : A rhombus is a parallelogram with all sides equal.

ABCD is a rhombus, diagonals AC and BD intersect at O

We want to show that AC is perpendicular to BD



Consider ΔAOB and ΔBOC

$OA = OC$ (Diagonals of a parallelogram bisect each other)

$OB = OB$ (common side to ΔAOB and ΔBOC)

$AB = BC$ (sides of rhombus)

Therefore $\Delta AOB \cong \Delta BOC$ (S.S.S rule)

So $\angle AOB = \angle BOC$

But $\angle AOB + \angle BOC = 180^\circ$ (Linear pair)

Therefore $2\angle AOB = 180^\circ$

or $\angle AOB = \dfrac{180^\circ}{2} = 90^\circ$

Similarly $\angle BOC = \angle COD = \angle AOD = 90^\circ$ (Same angle)

Hence AC is perpendicular on BD

So, the diagonals of a rhombus are perpendicular to each other.

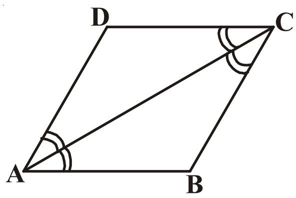
Corollary-3 : In a parallelogram ABCD, if the diagonal AC bisects the angle A, then ABCD is a rhombus.

Proof : ABCD is a parallelogram

Therefore AB || DC. AC is the transversal intersects $\angle A$ and $\angle C$

So, $\angle BAC = \angle DCA$ (Interior alternate angles) ...(1)

$\angle BCA = \angle DAC$ ...(2)



But it is given that AC bisects $\angle A$

So $\angle BAC = \angle DAC$

$\therefore \angle DCA = \angle DAC$ ...(3)

Thus AC bisects $\angle C$ also

From (1), (2) and (3), we have

$\angle BAC = \angle BCA$

In ΔABC, $\angle BAC = \angle BCA$ means that $BC = AB$ (isosceles triangle)

But $AB = DC$ and $BC = AD$ (opposite sides of the parallelogram ABCD)

$\therefore\ AB = BC = CD = DA$

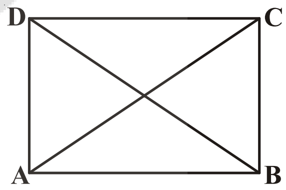
Hence, ABCD is a rhombus.

Corollary-4 : Show that the diagonals of a rectangle are of equal length.

Proof : ABCD is a rectangle and AC and BD are its diagonals

We want to know $AC = BD$

ABCD is a rectangle, means ABCD is a parallelogram with all its angles equal to right angle.



Consider the triangles ΔABC and ΔBAD,

$AB = BA$ (Common)

$\angle B = \angle A = 90^\circ$ (Each angle of rectangle)

$BC = AD$ (opposite sides of the rectangle)

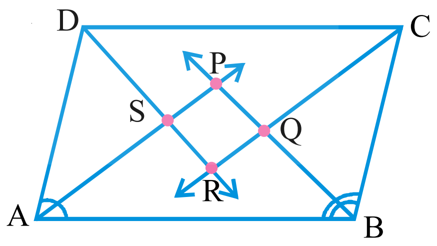
Therefore, $\Delta ABC \cong \Delta BAD$ (S.A.S rule)

This implies that $AC = BD$

or the diagonals of a rectangle are equal.

Corollary-5 : Show that the angle bisectors of a parallelogram form a rectangle.

Proof : ABCD is a parallelogram. The bisectors of angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$ intersect at P, Q, R, S to form a quadrilateral. (See adjacent figure)



Since ABCD is a parallelogram, $AD \parallel BC$. Consider AB as transversal intersecting them then $\angle A + \angle B = 180^\circ$(Consecutive angles of Parallelogram)

We know $\angle BAP = \frac{1}{2}\angle A$ and $\angle ABP = \frac{1}{2}\angle B$ [Since AP and BP are the bisectors of $\angle A$ and $\angle B$ respectively]

$$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{1}{2}\times 180^\circ$$

Or $\angle BAP + \angle ABP = 90^o$ ...(1)

But In ΔAPB,

$\angle BAP + \angle APB + \angle ABP = 180^\circ$ (Angle sum property of the triangle)

So $\angle APB = 180^\circ - (\angle BAP + \angle ABP)$

$\Rightarrow \angle APB = 180^\circ - 90^\circ$ (From (1))

$= 90^\circ$

We can see that $\angle SPQ = \angle APB = 90^\circ$

Similarly, we can show that $\angle CRD = \angle QRS = 90^\circ$ (Same angle)

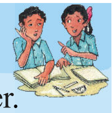
But $\angle BQC = \angle PQR$ and $\angle DSA = \angle PSR$ (Why?)

$\therefore$ $\angle PQR = \angle QRS = \angle PSR = \angle SPQ = 90^\circ$

Hence PQRS has all the four angles equal to 90°.

We can therefore say PQRS is a rectangle.

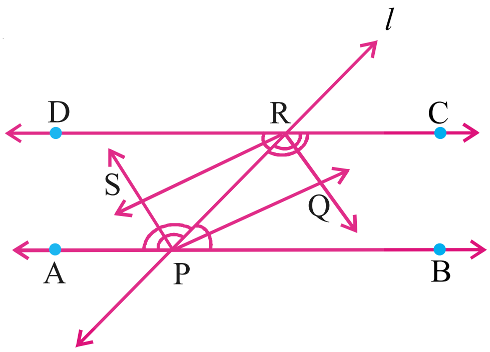
THINK, DISCUSS AND WRITE



1. Show that the diagonals of a square are equal and **right bisectors** of each other.
2. Show that the diagonals of a rhombus divide it into four congruent triangles.

Some Illustrative examples

Example-5. $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are two parallel lines and a transversal l, intersects $\overleftrightarrow{AB}$ at P and $\overleftrightarrow{DC}$ at R. Prove that the bisectors of the interior angles form a rectangle.



Proof : $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, l is the transversal intersecting $\overleftrightarrow{AB}$ at P and $\overleftrightarrow{DC}$ at R respectively.

Let $\overrightarrow{PQ}$, $\overrightarrow{RQ}$, $\overrightarrow{RS}$ and $\overrightarrow{PS}$ are the bisectors of $\angle RPB$, $\angle CRP$, $\angle DRP$ and $\angle APR$ respectively.

$\angle BPR = \angle DRP$ (Interior Alternate angles) ...(1)

But $\angle RPQ = \frac{1}{2}\angle BPR$ ($\because$ $\overrightarrow{PQ}$ is the bisector of $\angle BPR$)

also $\angle PRS = \frac{1}{2}\angle DRP$ ($\because$ $\overrightarrow{RS}$ is the bisector of $\angle DPR$). ...(2)

From (1) and (2)

$\angle RPQ = \angle PRS$

These are interior alternate angles made by $\overline{PR}$ with the lines $\overrightarrow{PQ}$ and $\overrightarrow{RS}$

$\therefore \quad \overrightarrow{PQ} \parallel \overrightarrow{RS}$

Similarly

$\angle PRQ = \angle RPS$, hence $\overrightarrow{PS} \parallel \overrightarrow{RQ}$

Therefore PQRS is a parallelogram ... (3)

We have $\angle BPR + \angle CRP = 180^o$ (interior angles on the same side of the transversal l with line $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$)

$\frac{1}{2}\angle BPR + \frac{1}{2}\angle CRP = 90^o$

$\Rightarrow \angle RPQ + \angle PRQ = 90^o$

But in Δ PQR,

$\angle RPQ + \angle PQR + \angle PRQ = 180^o$ (three angles of a triangle)

$\angle PQR = 180^o - (\angle RPQ + \angle PRQ)$

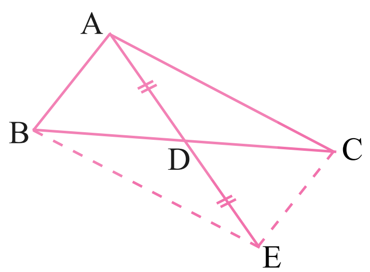
$= 180^o - 90^o = 90^o$ ... (4)

From (3) and (4)

PQRS is a parallelogram with one of its angles as a right angle.

Hence PQRS is a rectangle

Example-6. In a triangle ABC, AD is the median drawn on the side BC is produced to E such that AD = ED prove that ABEC is a parallelogram.



Proof : AD is the median of Δ ABC

Produce AB to E such that AD = ED

Join BE and CE.

Now In Δ^s ABD and ECD

BD = DC (D is the midpoints of BC)

$\angle ADB = \angle EDC$ (vertically opposite angles)

AD = ED (Given)

So $\Delta ABD \cong \Delta ECD$ (SAS rule)

Therefore, AB = CE (CPCT)

also $\angle ABD = \angle ECD$

There are interior alternate angles made by the transversal $\overleftrightarrow{BC}$ with lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CE}$.

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CE}$

Thus, in a Quadrilateral ABEC,

AB || CE and AB = CE

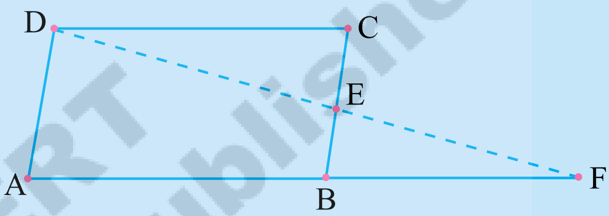
Hence ABEC is a parallelogram.

EXERCISE - 8.3

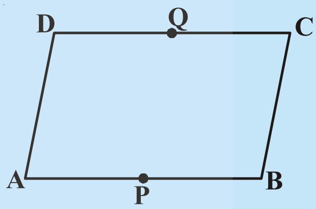
1. The opposite angles of a parallelogram are $(3x - 2)^o$ and $(x + 48)^o$. Find the measure of each angle of the parallelogram.

2. Find the measure of all the angles of a parallelogram, if one angle is 24^o less than the twice of the smallest angle.

3. In the adjacent figure ABCD is a parallelogram and E is the midpoint of the side BC. If DE and AB are produced to meet at F, show that AF = 2AB.



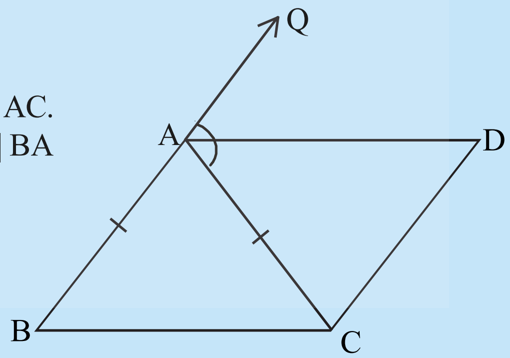
4. In the adjacent figure ABCD is a parallelogram P, Q are the midpoints of sides AB and DC respectively. Show that PBCQ is also a parallelogram.



5. ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle QAC and CD || BA as shown in the figure. Show that

   (i) $\angle DAC = \angle BCA$

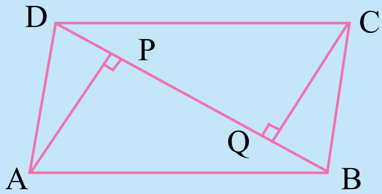
   (ii) ABCD is a parallelogram



6. ABCD is a parallelogram AP and CQ are perpendiculars drawn from vertices A and C on diagonal BD (see figure) show that

   (i) $\Delta APB \cong \Delta CQD$

   (ii) AP = CQ



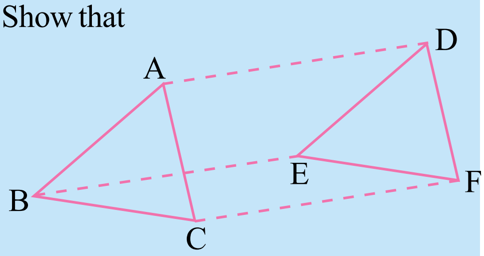
7. In Δ^s ABC and DEF, AB || DE; BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that

   (i) ABED is a parallelogram

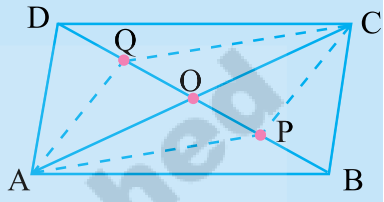
   (ii) BCFE is a parallelogram

   (iii) AC = DF

   (iv) $\Delta ABC \cong \Delta DEF$



8. ABCD is a parallelogram. AC and BD are the diagonals intersect at O. P and Q are the points of tri section of the diagonal BD. Prove that CQ || AP and also AC bisects PQ (see figure).



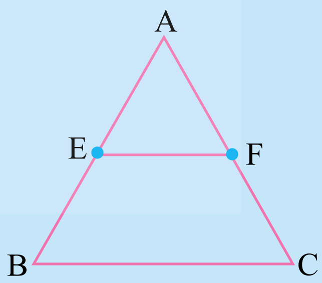
9. ABCD is a square. E, F, G and H are the mid points of AB, BC, CD and DA respectively. Such that AE = BF = CG = DH. Prove that EFGH is a square.

8.6 The Midpoint Theorem of Triangle

We have studied properties of triangle and of a quadrilateral. Let us try and consider the midpoints of the sides of a triangle and what can be derived from them.

Try This

Draw a triangle ABC and mark the midpoints E and F of two sides of triangle. $\overline{AB}$ and $\overline{AC}$ respectively. Join the point E and F as shown in the figure.



Measure EF and the third side BC of the triangle. Also measure $\angle AEF$ and $\angle ABC$.

We find $\angle AEF = \angle ABC$ and $\overline{EF} = \frac{1}{2}\overline{BC}$

As these are corresponding angles made by the transversal AB with lines EF and BC, we say EF || BC.

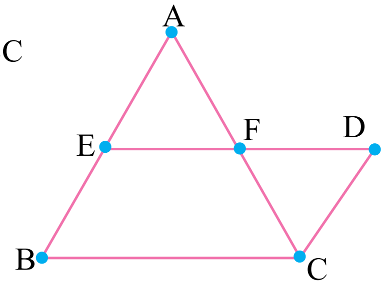
Repeat this activity with some more triangles.

So, we arrive at the following theorem.

Theorem-8.7 : The line segment joining the midpoints of two sides of a triangle is parallel to the third side and also half of it.

Given : ABC is a triangle with E and F as the midpoints of AB and AC respectively.

We have to show that : (i) EF || BC (ii) $EF = \frac{1}{2}BC$



Proof:- Join EF and extend it, and draw a line parallel to BA through C to meet to produced EF at D.

In Δ^s AEF and ΔCDF

AF = CF (F is the midpoint of AC)

$\angle AFE = \angle CFD$ (vertically opposite angles.)

and $\angle AEF = \angle CDF$ (Interior alternate angles as CD || BA with transversal ED.)

By A.S.A congruency rule

$\therefore \Delta AEF \cong \Delta CDF$ ASA congruency rule

Thus AE = CD and EF = DF (CPCT)

We know AE = BE

Therefore BE = CD

Since BE || CD and BE = CD, BCDE is a parallelogram.

So ED || BC

$\Rightarrow$ EF || BC

As BCDE is a parallelogram, ED = BC(how ?) ($\because$ DF = EF)

But we have shown FD = DF

$\therefore$ 2EF = BC

Hence $EF = \frac{1}{2}BC$

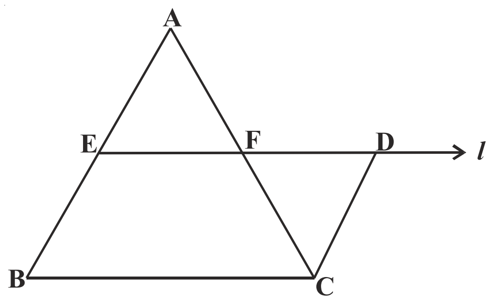
We can see that the converse of the above statement is also true. Let us state it and then see how we can prove it.

Theorem-8.8 : The line drawn through the midpoint of one of the sides of a triangle and parallel to another side will bisect the third side

Proof: Draw ΔABC. Mark E as the mid point of side AB. Draw a line *l* passing through E and parallel to BC. The line intersects AC at F.

Construct CD || BA

We have to show AF = CF



Consider ΔAEF and ΔCFD

$\angle EAF = \angle DCF$ (BA || CD and AC is transversal) (How ?)

$\angle AEF = \angle D$ (BA || CD and ED is transversal) (How ?)

We can not prove the congruence of the triangles as we have not shown any pair of sides in the two triangles as equal.

To do so we consider EB || DC

ED || AC

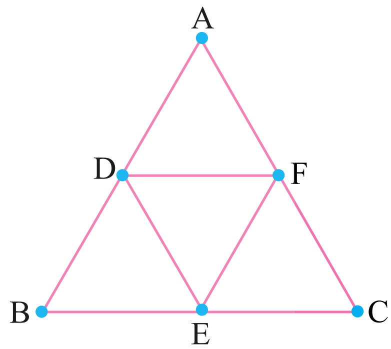
Thus EDCB is a parallelogram and we have BE = DC.

Since BE = AE we have AE = DC.

Hence $\Delta AEF \cong \Delta CFD$

$\therefore$ AF = CF

Some more examples



Example-7. In ΔABC, D, E and F are the midpoints of sides AB, BC and CA respectively. Show that ΔABC is divided into four congruent triangles, when the three midpoints are joined to each other. (ΔDEF is called medial triangle)

Proof : D, E are midpoints of $\overline{AB}$ and $\overline{AC}$ of triangle ABC respectively

so by Mid-point Theorem,

DE || AC

Similarly DF || BC and EF || AB.

Therefore ADEF, BEFD, CFDE are all parallelograms

In the parallelogram ADEF, DF is the diagonal

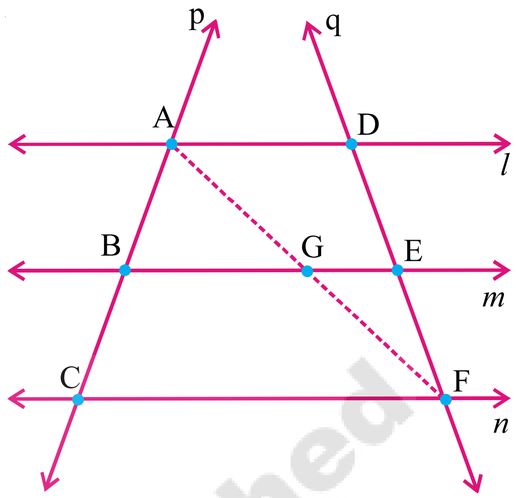
So $\Delta ADF \cong \Delta DEF$ (Diagonal divides the parallelogram into two congruent triangles)

Similarly $\Delta BDE \cong \Delta DEF$

and $\Delta CEF \cong \Delta DEF$

So, all the four triangles are congruent.

We have shown that a triangle ABC is divided in to four congruent traingles by joining the midpoints of the sides.



Example-8. l, m and n are three parallel lines intersected by the transversals p and q at A, B, C and D,E, F such that they make equal intercepts AB and BC on the transversal p. Show that the intercepts DE and EF on q are also equal.

Proof : We need to connect the equality of AB and BC to comparing DE and EF. We join A to F and call the intersection point with 'm' as G.

In ΔACF, AB = BC (given)

Therefore B is the midpoint of AC.

and BG || CF (how ?)

So G is the midpoint of AF (By the theorem).

Now in Δ AFD, we can apply the same reason as G is the midpoint of AF and GE || AD, E is the midpoint of DF.

Thus DE = EF.

Hence l, m and n cut off equal intersects on q also.

Example-9. In the Fig. AD and BE are medians of ΔABC and BE || DF. Prove that

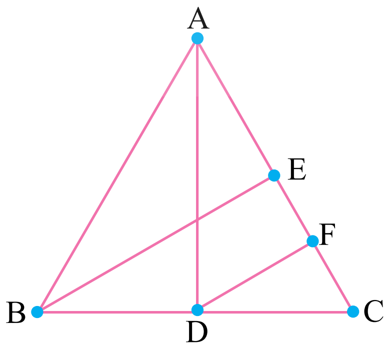
$$CF = \frac{1}{4}AC.$$

Proof : If Δ ABC, D is the midpoint of BC and BE || DF; By Theorem F is the midpoint of CE.

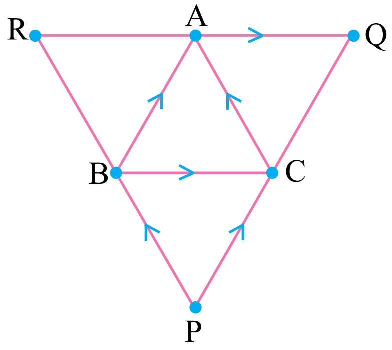
$$\therefore CF = \frac{1}{2}CE$$

$$= \frac{1}{2}\left(\frac{1}{2}AC\right) \text{ (How ?)}$$

Hence $CF = \frac{1}{4}AC$.



Example-10. ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.



Proof : AB || QP and BC || RQ So ABCQ is a parallelogram.

Similarly BCAR, ABPC are parallelograms

$\therefore$ BC = AQ and BC = RA

$\Rightarrow$ A is the midpoint of QR

Similarly B and C are midpoints of PR and PQ respectively.

$\therefore AB = \frac{1}{2}PQ; \quad BC = \frac{1}{2}QR$ and $CA = \frac{1}{2}PR$ (How)

(State the related theorem)

Now perimeter of $\Delta PQR = PQ + QR + PR$

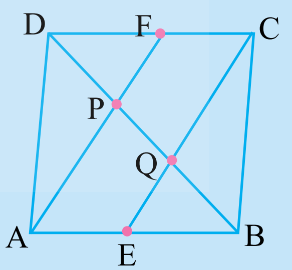
$= 2AB + 2BC + 2CA$

$= 2(AB + BC + CA)$

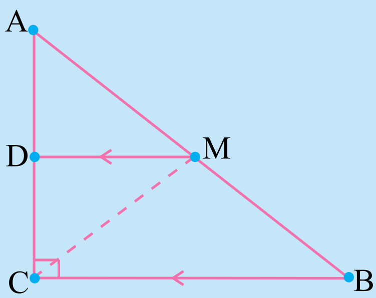
$= 2$ (perimeter of ΔABC).

EXERCISE - 8.4

1. ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4}AB$ and E is a point on AC such that $AE = \frac{1}{4}AC$. If DE = 2 cm find BC.
2. ABCD is quadrilateral E, F, G and H are the midpoints of AB, BC, CD and DA respectively. Prove that EFGH is a parallelogram.
3. Show that the figure formed by joining the midpoints of sides of a rhombus successively is a rectangle.
4. In a parallelogram ABCD, E and F are the midpoints of the sides AB and DC respectively. Show that the line segments AF and EC trisect the diagonal BD.



5. Show that the line segments joining the midpoints of the opposite sides of a quadrilateral and bisect each other.
6. ABC is a triangle right angled at C. A line through the midpoint M of hypotenuse AB and Parallel to BC intersects AC at D. Show that
   (i) D is the midpoint of AC
   (ii) MD $\perp$ AC
   (iii) $CM = MA = \frac{1}{2}AB$.

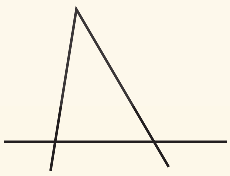


WHAT WE HAVE DISCUSSED

1. A quadrilateral is a simple closed figure formed by four lines in a plane.
2. The sum of four angles in a quadrilateral is 360^0 or 4 right angles.
3. Trapezium, parallelogram, rhombus, rectangle, square and kite are special types of quadrilaterals
4. Parallelogram is a special type of quadrilateral with many properties. We have proved the following theorems.
   a) The diagonal of a parallelogram divides it into two congruent triangles.
   b) The opposite sides and angles of a parallelogram are equal.
   c) If each pair of opposite sides of a quadrilateral are equal then it is a parallelogram.
   d) If each pair of opposite angles are equal then it is a parallelogram.
   e) Diagonals of a parallelogram bisect each other.
   f) If the diagonals of a quadrilateral bisect each other then it is a parallelogram.
5. Mid point theorm of triangle and converse
   a) The line segment joining the midpoints of two sides of a triangle is parallel to the third side and also half of it.
   b) The line drawn through the midpoint of one of the sides of a triangle and parallel to another side will bisect the third side.

Brain teaser

1. Creating triangles puzzle



Add two straight lines to the above diagram and produce 10 triangles.

2. Take a rectangular sheet of paper whose length is 16 cm and breadth is 9 cm. Cut it in to exactly 2 pieces and join them to make a square.